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Habit Formation, Catching Up with the Joneses, and Economic Growth*

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Our objective is to investigate how alternative assumptions about preferences affect the process of economic growth. To do this, we analyze a neoclassical growth model under three alternative preference specifications: (i) time separable, (ii) catching up with the Joneses, and (iii) habit formation. Departing from the time separable specification leads to important differences in the dynamic structure, the adjustment path followed by key economic variables, the correlation patterns implied by the time series generated by the model, and the speed of convergence to the new steady state. In the catching up with the Joneses economy the differences arise from a consumption externality, while in the habit formation economy the difference arises from the fact that agents not only smooth consumption but also its rate of change.

Keywords: habit formation, consumption externalities, economic growth

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1. Introduction

The concepts of habit and status have long been acknowledged as being important characteristics of human behavior. The idea that the overall level of satisfaction derived from a given level of consumption depends, not only on the (current) consumption level itself, but also on how it compares with some benchmark level, is not new. Origins of this proposition can be traced as far back as Smith (1759) and Veblen (1899), although it was

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not until Duesenberry (1949) that an effort was made to provide these ideas with some micro-theoretic foundations.

Subsequent literature has identified two types of reference consumption levels that may characterize these ''time non-separable'' preference functions. The first is based on an external criterion, expressed in terms of the past consumption of some outside reference group, typically the average consumption of the overall economy. This is often referred to as ''catching up with the Joneses'' or ''utility-interdependence'', and the agent described as being ''outward-looking''. The second is an internal criterion based on the individual's own past consumption levels. It is often referred to as characterizing ''habit formation'', and the agent described as being ''inward-looking''.

A growing body of empirical evidence has confirmed the importance of time non-separable preferences. Using panel data for the Netherlands, Van de Stadt et al. (1985) model both habit formation and utility interdependence. Their results are compatible with the hypothesis that utility depends upon relative consumption, although they cannot exclude the possibility that utility reflects both relative and absolute consumption. Using U.K. data, Osborn (1988) introduces a consumption specification that allows for seasonal variations and habit persistence, and finds the habit persistent terms to be jointly significant. More recently, Fuhrer (2000) strongly rejects the hypothesis of time separable preferences. Employing a utility function that assigns relative weights to both current consumption and an internal benchmark, he finds 80 percent of the weight should be attached to the latter.[1] In addition, Fuhrer and Klein (1998) present empirical evidence suggesting that habit formation is a relevant characteristic of consumption behavior among the G-7 countries.

An extensive literature on asset-pricing anomalies, most notably the equity premium puzzle, lends further credence to the level of benchmark consumption being a significant determinant of consumption behavior. Habit-forming consumers dislike large and rapid cuts in consumption. As a result, the premium that they will require to hold risky assets that might force a rapid cut in consumption will be large relative to that implied by the time-separable utility model. This feature of time non-separable preferences is exploited by Abel (1990), Constantinides (1990), Gali (1994), and Campbell and Cochrane (1995) among others.

Despite this evidence supporting the relevance of benchmark consumption levels for current consumption decisions, relatively few attempts have been made to introduce time non-separable preferences into the growth literature, where the specification of preferences as time-separable functions remains standard. One notable early exception is Ryder and Heal (1973), who introduced habit formation into the basic neoclassical growth model. The focus of their paper is to study the role of habit formation in determining the generic nature of the transitional adjustment path, rather than in analyzing how habit formation influences the impact of structural changes on the evolution of the economy. More recently, this approach has been pursued by Carroll et al. (1997, 2000),

1 However, his evidence is inconclusive with respect to the weights assigned to past consumption levels in forming the benchmark level. He cannot reject the hypothesis that it is completely determined by the previous period's consumption.

Fisher and Hof (2000), Alonso-Carrera et al. (2001a,b), although mostly under rigid production conditions that characterize the simplest endogenous growth model.[2]

However, time separable utility may yield misleading conclusions if in fact preferences are characterized by a high degree of complementarity between consumption at successive moments, as the empirical evidence suggests. Thus, given the acknowledged limitations of the endogenous growth model, it is important to analyze further the role of interdependent preferences under more flexible production conditions.[3] To do so is the objective of the present paper. Specifically, we consider the implications of time non-separable preferences using a one-sector neoclassical growth model in which labor experiences positive productivity growth.[4] Using this model, we analyze the dynamic responses to two contrasting types of shocks: (i) an increase in the rate of productivity growth, and (ii) a destruction in the initial stock of capital.

Of the studies cited, our analysis is closest to Carroll et al. (1997, 2000). One of their objectives was to compare the introduction of time non-separable preferences with traditional (time-separable) preferences, and to isolate the role of preferences they intentionally restrict the production side to the simplest possible form. Thus they show that whereas with conventional preferences the basic AK technology they employ always places the economy on its balanced growth path, the introduction of time non-separable preferences introduces sluggishness into the system, so that the economy approaches its balanced growth equilibrium along a transitional path.

But whether the production function has diminishing rather than constant returns to capital has important consequences. First, the equilibrium transitional adjustment paths may now exhibit non-monotonic behavior, something that is not possible under the simple AK technology, but which nevertheless is important in replicating certain observed stylized facts. Second, the implied adjustments to the two shocks considered in this paper, are qualitatively quite different, depending upon whether the productivity of capital is constant or diminishing.

While Carroll et al. (1997, 2000) is a significant contribution, at least in one important case the implied transitional dynamics are inconsistent with available empirical evidence. Specifically, they consider the consequences of a destruction in the initial capital stock. They find that, whereas with traditional preferences this has no impact on the growth rate, with time non-separable preferences (both inward- and outward-looking) it involves an initial reduction in the growth rate, which then gradually increases during transition and eventually returns to its pre-shock level. In addition, the savings rate decreases on impact, and thereafter gradually increases monotonically over time.

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2 An exception is Alonso-Carrera et al. (2001b), who employ the hybrid neoclassical-AK production function introduced by Jones and Manuelli (1990).

3 Despite its many appealing features the endogenous growth model has drawn sharp criticisms. For example, Solow (1994) criticizes the constraints that this model imposes on the underlying technologies. Jones (1995) and Backus et al. (1992) criticize some of the empirical implications, involving ''scale effects'' that are not supported by the data.

4 The analysis easily extends to the more general non-scale technology studied by Eicher and Turnovsky (1999a,b). In an earlier version of this paper, we have introduced time non-separable preferences into such a model having non-constant returns to scale.

But experience pertaining to the destruction of capital in both Europe and Japan during World War II suggests that precisely the opposite adjustments in fact occurred. Studies by Maddison (1994) and Wolf (1993) suggest that around 30 percent of the capital stock in Germany was destroyed between 1939 and 1945, while Saint-Paul (1993) estimates the war time destruction of capital in France to be between 20 and 35 percent. Data compiled from Maddison (2001) suggests that average growth rates in European economies jumped to over 7 percent in 1950, and then gradually declined to around 4 percent by 1970. Moreover, data from Maddison (1992) shows that the average European savings rate increased from just under 22 percent in 1950, peaking at over 28 percent in 1961, before slightly declining over the next decade.[5] Similar patterns for the growth rate and savings rate are documented by Christiano (1989) in the case of Japan; see also King and Rebelo (1993).

One of the main findings of this paper is that introducing time non-separable preferences, in conjunction with the neoclassical technology and the more flexible transitional dynamic adjustment paths it permits, can easily generate time paths for the growth rate and the savings rate during the early stages of the transition following an initial loss in the capital stock that will replicate this observed non-monotonic behavior. Our analysis emphasizes how the transitional dynamics are driven by two opposing forces, one originating with preferences—what we call the ‘‘status effect’’—the other arising from the diminishing returns to capital. By restricting themselves to an AK production technology, Carroll et al. (1997, 2000) incorporate only the former effect. This substantially restricts the dynamic behavior of the system to monotonic adjustments paths driven largely by preference parameters. Thus, one of the general conclusions we draw is the potential importance of combining (i) more general preferences with (ii) a more flexible production technology, in replicating observed behavior. But the fact that our equilibrium can generate more flexible dynamic paths comes at a price. This is because the added flexibility reflects a higher order dynamic system that is too intractable to be studied analytically, but instead must be analyzed using numerical simulations.

We should note that the non-monotonic transitional adjustment paths emphasized in this model could also be obtained using the hybrid technology employed by Alonso-Carrera et al. (2001b), although their focus was on characterizing the equilibrium and efficiency issues, rather than analyzing transitional adjustment paths. In this respect, our formulation differs in an important way. Alonso-Carrera et al. (2001a,b) assume that the reference stock in utility is determined by the previous period’s consumption; thus it adjusts rapidly. Hence, any empirical regularity that requires slow-moving habits cannot be explained by their specification. The behavior of saving after World War II is one such case, and to replicate it within this class of model we need a slow-moving reference stock.

We employ the utility function introduced by Abel (1990) in the context of asset pricing and used by Carroll et al. (1997, 2000). Following these authors we shall consider both externally and internally generated consumption benchmarks. We shall compare their implications for the dynamic adjustment of the neoclassical growth model, both to one another as well as to those of the conventional time-separable specification of preferences.

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5 This empirical evidence is discussed and documented in greater detail by Alvarez-Cuadrado (2003).

Departing from the basic growth model in the specification of preferences yields important differences in the equilibrium dynamics, the adjustment process of key economic variables, the correlation patterns implied by the model, and the speed of convergence to the new steady state.

There are several key results that we wish to stress at the outset. The first and most general finding is that the differences between assuming the conventional time-separable utility function, on the one hand, and time non-separable preference functions, on the other, are substantial. By contrast, the difference between assuming that the reference consumption level is formed by looking outwards or inwards is relatively small, although it does depend upon the shock imposed upon the economy.[6]

Second, in contrast to the AK model, introducing time non-separable utility may increase, rather than decrease, the speed of convergence. This depends upon how rapidly the reference stock adjusts relative to the intrinsic adjustment speed in the rest of the economy. Third, the introduction of consumption habits causes substantial intertemporal shifts in the time paths for consumption and savings following structural shocks to the economy. In the case of an increase in the productivity growth rate it leads to a smaller short-run increase in consumption and a larger increase in saving, which over time generates an eventual larger increase in consumption. The impact of habit is even more dramatic in the case where the shock takes the form of a destruction of capital. Fourth, the time path of welfare resulting from a structural change can be decomposed into the effect on the absolute consumption level, together with the effect on current consumption relative to the reference level. This can lead to substantially different welfare implications from those obtained for conventional preferences, depending upon how rapidly the reference stock is assumed to adjust. Consequently, the policy and welfare implications of structural changes, conducted under the conventional assumption of time-separable preferences may turn out to be quite misleading if in fact preferences are time non-separable. Fifth, the initial stages of the dynamics are particularly sensitive to the speed of adjustment of the reference consumption level; they are less sensitive to the weight assigned to the reference consumption level in utility. Sixth, the presence of a reference consumption level can have a very different effect on the transitional dynamics in a neoclassical model from its effect in the endogenous growth model. This depends upon how non-monotonic the transitional paths are in the former, which in part is sensitive to the adjustment speed of habits. Finally, time non-separable preferences provide interesting insight into the growth-saving relation. In contrast to the conventional model where saving is seen as the engine of growth, our model reverses this causal relation, suggesting that growth leads to saving. This behavior is consistent with the European evidence cited earlier, as well as the more formal causality tests conducted by Carroll and Weil (1994) and Attanasio et al. (2000).

The paper is organized as follows. Section 2 sets out the basic structure of the model, introducing our two versions of time non-separable preferences. Section 3 then characterizes the corresponding macroeconomic behavior of the economy. Section 4 conducts a numerical analysis, comparing the dynamic responses of the economy under

6 It also depends upon the assumption that labor is supplied inelastically.

the alternative specifications of preferences, while Section 5 carries out some sensitivity analysis. Section 6 compares the implications of the present neoclassical model with those obtained under the more restrictive AK production structure. The conclusions are summarized in Section 7, while an Appendix provides some technical details.

2. The Model

Consider an economy populated by N identical and infinitely lived households that grows at the exogenous rate $\dot{N}/N = n$. At any point in time, households derive utility from the comparison of their current consumption level relative to a reference consumption level. The individual household's objective is to maximize the intertemporal iso-elastic utility function:

$$\Omega \equiv \frac{1}{1-\varepsilon}\int_0^\infty \left[\frac{C_i}{H_i^\gamma}\right]^{1-\varepsilon} \mathrm{e}^{-\beta t} dt = \frac{1}{1-\varepsilon}\int_0^\infty \left[C_i^{(1-\gamma)}\left(\frac{C_i}{H_i}\right)^\gamma\right]^{1-\varepsilon} \mathrm{e}^{-\beta t} dt, \tag{1}$$

where C_i and H_i are household i's current consumption and reference consumption level (habits), respectively.[7] Following Ryder and Heal (1973) we impose non-satiation in utility, restricting γ to lie in the range $0 \le \gamma < 1$.[8] As we can see from the second expression in (1), agents derive utility from a geometric weighted average of absolute and relative consumption, these corresponding to the polar cases, $\gamma = 0$, and $\gamma \to 1$, respectively. In general, ε and γ interact to determine the (consumption) intertemporal elasticity of substitution (IES), having the property that it varies with the horizon considered. From (2) we see that as the time horizon shrinks to zero, habits are predetermined and fixed, and therefore the IES converges to the expression for the conventional time-separable case, $1/\varepsilon$. At the other extreme, as the time horizon increases to infinity, habits fully adjust to a change in consumption. Setting $H_i = C_i$ in (1) this implies a long-run intertemporal elasticity of substitution equal to $1/(\gamma + \varepsilon(1-\gamma))$.[9] This contrasts with the conventional case, $\gamma = 0$, where the intertemporal elasticity of substitution, $1/\varepsilon$, remains constant and independent of the horizon considered. Thus we see that the long-run IES under time non-separable preferences exceeds the conventional IES if and only if $1/\varepsilon < 1$, as empirical evidence overwhelmingly suggests.

The agent's reference stock is assumed to be specified by

7 The form of the utility specification (1) raises questions about whether or not the necessary conditions that we derive are in fact optimal. This problem is characteristic of all the literature that employs the utility function in (1). In the case of outward-looking agents, H_i is an externality and the utility function is concave in C_i. Given that the constraints are concave functions, the first-order conditions suffice to ensure a maximum. By contrast, in the case of inward-looking agents (habit formation), the utility function is not jointly concave in both C_i and H_i, and thus the first-order conditions may not yield a maximum. In this case, the paper by Alonso-Carrera et al. (2001b) argues that the interior solution will ensure utility maximization if one restricts $\varepsilon > 1$, consistent with the empirical evidence.

8 Non-satiation is guaranteed if an increase in a uniformly maintained consumption level increases utility, that is, if $U_C(C_i, C_i) + U_H(C_i, C_i) > 0$.

9 See also Carroll et al. (2000).

$$H_i(t) = \rho \int_{-\infty}^{t} e^{\rho(\tau - t)} C_i(\tau)^{\phi} \overline{C}(\tau)^{1-\phi} d\tau \quad 0 \leq \phi \leq 1 \;\; \rho > 0, \tag{2}$$

where $\overline{C} = \Sigma_{i=1}^{N} C_i/N$ denotes the economy-wide average consumption of agents. Equations (1) and (2) encompass the three specifications of preferences that we wish to consider, being identified by different values of the parameters γ and ϕ. The benchmark conventional preferences are obtained by setting $\gamma = 0$ in (1), in which case the reference stock as represented by (2) is irrelevant. Setting $\phi = 0$ identifies the outward-looking agent, for whom the reference stock is formed as an exponentially declining weighted average of past economy-wide average levels of consumption. Since agents are atomistic, they ignore the effect of their individual consumption decisions on the evolution of the reference stock, taking it as exogenous. Setting $\phi = 1$ corresponds to the inward-looking agent, for whom the reference stock is an exponentially declining weighted average of his own past levels of consumption.

The differences in behavior between the latter two cases arise from the fact that in the outward-looking economy the reference stock is an externality, whereas in the inward-looking economy it is not. The outward-looking agent ignores the effect that his present consumption induces on his future utility through its effect on the average consumption. In contrast, the inward-looking agent fully internalizes the effect of his current consumption decision on the future evolution of his reference stock and thus on his future welfare.

Differentiating (2) with respect to time implies the following rate of adjustment for the reference stock

$$\dot{H}_i = \rho\left(C_i^{\phi}\overline{C}^{1-\phi} - H_i\right). \tag{3}$$

The speed of adjustment, ρ, parameterizes the relative importance of recent consumption in determining the reference stock. Therefore, higher values of ρ lead to a higher influence of current consumption in the determination of the future reference stock, or alternatively to a lower level of persistence in habits.

Individual output is determined by the agent's capital stock, K_i, and his level of inelastically supplied labor, L_i. We shall assume that labor productivity grows at the exogenous constant rate, $\dot{A}/A = g$. Assuming a Cobb–Douglas production function, individual output is determined by,

$$Y_i = \alpha(AL_i)^{\sigma} K_i^{(1-\sigma)} \quad 0 < \sigma < 1. \tag{4}$$

The technology exhibits diminishing marginal product to each private factor and constant returns to scale in the two factors, capital and labor in efficiency units.

Final output can be either consumed currently, or saved and transformed into additional capital to yield future consumption. Assuming that the existing capital stock depreciates at a rate, δ, agent i's capital stock evolves according to the accumulation relationship

$$\dot{K}_i = Y_i - C_i - (n + \delta)K_i. \tag{5}$$

The agent chooses his consumption, rate of capital accumulation, and rate of change of the reference stock to maximize (1), subject to the production function (4), the accumulation equation (5), and the evolution of the reference stock (3). This yields the optimality conditions

$$\frac{C_i^{-\varepsilon}}{H_i^{\gamma(1-\varepsilon)}} + \rho\phi\lambda_{2i}C_i^{\phi-1}\overline{C}^{1-\phi} = \lambda_{1i}, \tag{6a}$$

$$\alpha(1-\sigma)\frac{Y_i}{K_i} - \delta - n = \beta - \frac{\dot{\lambda}_{1i}}{\lambda_{1i}}, \tag{6b}$$

$$U_{H_i} \equiv -\gamma\frac{C_i^{(1-\varepsilon)}}{H_i^{\gamma(1-\varepsilon)+1}} = \lambda_{2i}(\beta+\rho) - \dot{\lambda}_{2i}, \tag{6c}$$

where λ_{1i} denotes the agent's shadow value of capital, λ_{2i} is the shadow value of the agent's reference stock, together with the transversality conditions

$$\lim_{t\to\infty} \mathrm{e}^{-\beta t}\lambda_{1i}K_i = \lim_{t\to\infty} \mathrm{e}^{-\beta t}\lambda_{2i}H_i = 0. \tag{6d}$$

Equation (6a) equates the utility of an additional unit of consumption adjusted by its impact on the future reference stock to the shadow value of capital, while equation (6b) is the standard intertemporal allocation condition equating the marginal product of capital to the rate of return on consumption. Equation (6c) is an additional intertemporal allocation condition equating the marginal disutility of an additional unit of habit measured in terms of its shadow value to the cost of habit.

Equations (6) nest the optimality conditions for the three hypotheses regarding preferences that we wish to consider. Internal preferences ($\phi = 1$) require the monitoring of two state variables, and this continues to apply as long as some positive weight is assigned to the agent's own past consumption in his reference stock ($\phi > 0$). In the case of a purely outward-looking consumption benchmark ($\phi = 0$), λ_{2i} is irrelevant, although the agent still needs to take account of the fact that utility depends upon current consumption relative to the external benchmark. In the conventional case of time-separable preferences ($\gamma = 0$), (6c) implies $\lambda_{2i} \equiv 0$, in which case (6a) and (6b) reduce to the usual optimality conditions, and the evolution of the benchmark consumption level becomes irrelevant. The equilibrium dynamics corresponding to these three cases will be discussed below.

3. Macroeconomic Equilibrium

We now proceed to derive the macroeconomic equilibria. With all individuals being identical, aggregate capital, output, consumption, and the reference consumption stock are given by $K \equiv NK_i$, $Y \equiv NY_i$, $C \equiv NC_i$, and $H \equiv NH_i$, respectively. Normalizing L_i to 1, and summing across households, yields the aggregate production function,

$$Y = \alpha(AN)^{\sigma}K^{1-\sigma}. \tag{7}$$

We define a balanced growth path as being one along which all variables grow at a constant rate. With capital being accumulated from final output, along a balanced growth path K/Y

remains constant. Differentiating the aggregate production function (7), we obtain the long-run equilibrium growth rates of capital and output, $\hat{K}^*$ and $\hat{Y}^*$,

$$\hat{Y}^* = \hat{K}^* = g + n, \tag{8}$$

yielding the standard property that the equilibrium growth rate equals the population growth rate plus the exogenous growth rate of labor productivity.

Following our definition of the balanced growth path, it is convenient to write the system in terms of the following normalized variables $k \equiv K/AN$, $y \equiv Y/AN$, $c \equiv C/AN$, and $h \equiv H/AN$, all expressed in units of effective labor, which remain constant in steady-state equilibrium. In addition, with all agents being identical, we may drop the subscript i on the shadow values and consider $q \equiv \lambda_{2i}/\lambda_{1i}$, which also is stationary.

Taking the time derivative of (6a), combining with (6b) and (6c), imposing the equilibrium condition $C_i = \overline{C} = C/N$, and aggregating across households, the equilibrium path for the growth rate of aggregate consumption is,

$$\hat{C} \equiv \frac{\dot{C}}{C} = \frac{1}{\varepsilon}\left[\frac{(1-\sigma)}{1-\rho q\phi}\alpha\left(\frac{K}{AN}\right)^{-\sigma} + \rho\gamma\left(\frac{C}{H}\right)[\phi-(1-\varepsilon)] + \gamma\rho(1-\varepsilon) + \frac{\rho q\phi(\beta+\rho)-\delta-\beta-n}{1-\rho q\phi} + n\varepsilon\right], \tag{9}$$

which describes the evolution of aggregate consumption in terms of the productivity of capital and the ratio of consumption to the reference stock.

Taking the time derivatives of k, c, h, and q and combining these definitions with (3), (5), and (9), the dynamic behavior of the economy can be described by the following system of differential equations in k, c, h, and q:

$$\dot{k} = \alpha k^{1-\sigma} - c - (\delta + n + g)k, \tag{10a}$$

$$\dot{c} = \frac{c}{\varepsilon}\left\{\frac{(1-\sigma)}{1-\rho q\phi}\alpha k^{-\sigma} + \rho\gamma\frac{c}{h}[\phi-(1-\varepsilon)] + \gamma\rho(1-\varepsilon) + \frac{\rho q\phi(\beta+\rho)-\beta-\delta-n}{1-\rho q\phi} - \varepsilon g\right\}. \tag{10b}$$

$$\dot{h} = \rho(c-h) - gh, \tag{10c}$$

$$\dot{q} = q\left\{(1-\sigma)\alpha k^{-\sigma} + \gamma\frac{c}{h}\left(\frac{1}{q} - \rho\phi\right) + \rho - \delta - n\right\}. \tag{10d}$$

In the case of the inward-looking consumption benchmark, the dynamics of all four variables are interdependent. In contrast, if the benchmark is outward-looking, the dynamics decouple as follows. Setting $\phi = 0$, (10a)–(10c) form an autonomous sub-system in k, c, and h, but independent of q. Given the evolution of k, c, and h, the shadow value q then evolves in accordance with (10d), but is of no consequence insofar as the macrodynamic equilibrium is concerned. If $\gamma = 0$, so that the reference stock is irrelevant to utility, the dynamics decouple further. As noted, $\lambda_2 \equiv 0$ implying $q \equiv 0$. Now (10a) and (10b) jointly determine the evolution of k and c, independently of h, in the conventional way. Given the time path of c, (10c) then determines the evolution of h, but this is irrelevant insofar as the equilibrium is concerned.[10]

Imposing the stationary conditions, $\dot{c} = \dot{h} = \dot{q} = \dot{k} = 0$ in (10a)–(10d) we can determine the steady-state values of stationary variables in the following recursive manner. First, (10c) yields the consumption-habit ratio. Second, given c/h, (10d) determines the ratio of the shadow values, q, in terms of normalized capital, k. Third, substituting this stationary value of q into (10b) yields a quadratic equation in normalized capital, one of the roots of which can be eliminated by imposing the transversality condition (6d); see Appendix. Finally (10a) determines the steady-state level of normalized consumption. Letting tildes denote steady-state values, we may summarize these expressions as follows:

$$\tilde{k} = \left[\frac{\beta + \delta + n + (\gamma + \varepsilon(1-\gamma))g}{(1-\sigma)\alpha}\right]^{-1/\sigma}, \tag{11a}$$

$$\tilde{c} = \alpha(\tilde{k})^{1-\sigma} - (n + \delta + g)\tilde{k}, \tag{11b}$$

$$\frac{\tilde{c}}{\tilde{h}} = \frac{\rho + g}{\rho}, \tag{11c}$$

$$\tilde{q} = \frac{\gamma((g/\rho) + 1)}{\rho(\phi\gamma - 1) - \beta + (\gamma(\phi - 1) + \varepsilon(\gamma - 1))g}. \tag{11d}$$

The following observations about the steady state can be made. First, equations (11a–c) are independent of ϕ, implying that the steady-state values of normalized capital, consumption, and habits are the same whether the reference consumption level is formed internally or externally. Note from (11d) that $\tilde{q} < 0$. Intuitively, because an increase in the level of habits, given current consumption, is welfare-reducing, the shadow value of the reference stock is negative. Second, in the absence of productivity growth $(g = 0)$, (11c) implies $\tilde{c} = \tilde{h}$, so that the stationary consumption level coincides with the reference level. In that case (11a) reduces to the standard modified golden rule stock of capital, consistent with the early result of Ryder and Heal (1973). The equilibrium stock of capital will be independent of γ, the relative weight attributed to habit in utility, and will be the same for all specifications of preferences. Third, in the event of positive productivity growth, $(g > 0)$, $\tilde{c} > \tilde{h}$ by an amount that is inversely related to ρ, the speed with which the reference stock adjusts to recent consumption experience. In that case, the introduction of benchmark consumption into utility unambiguously increases the equilibrium normalized stock of capital if and only if $\varepsilon > 1$.

The dynamics can be approximated by the fourth-order system presented below:

10 Note that the dynamics of k and c also proceed independently if $\rho = 0$, so that the reference stock is fixed.

$$\begin{pmatrix} \dot{k} \\ \dot{c} \\ \dot{h} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} \alpha(1-\sigma)\tilde{k}^{-\sigma} - \delta - g - n & -1 & 0 & 0 \\ \frac{\alpha(\sigma-1)(\sigma)\tilde{k}^{-\sigma-1}}{1-\tilde{q}\rho\phi}\left(\frac{\tilde{c}}{\varepsilon}\right) & -\left(\frac{\tilde{c}}{\tilde{h}}\right)\left(\frac{\rho\gamma(1-\phi-\varepsilon)}{\varepsilon}\right) & \left(\frac{\tilde{c}}{\tilde{h}}\right)^2\left(\frac{\rho\gamma(1-\phi-\varepsilon)}{\varepsilon}\right) & \left(\frac{\rho\phi\tilde{c}}{\varepsilon}\right)\left[\frac{\alpha(1-\sigma)\tilde{k}^{-\sigma}+\rho-\delta-n}{(1-\rho\phi\tilde{q})^2}\right] \\ 0 & \rho & -\rho\left(\frac{\tilde{c}}{\tilde{h}}\right) & 0 \\ \alpha\tilde{q}(\sigma-1)(\sigma)\tilde{k}^{-\sigma-1} & \frac{\gamma}{\tilde{h}}(1-\rho\phi\tilde{q}) & -\gamma\frac{\tilde{c}}{\tilde{h}^2}(1-\rho\phi\tilde{q}) & -\left(\frac{\gamma}{\tilde{q}}\right)\left(\frac{\tilde{c}}{\tilde{h}}\right) \end{pmatrix} \begin{pmatrix} k-\tilde{k} \\ c-\tilde{c} \\ h-\tilde{h} \\ q-\tilde{q} \end{pmatrix}. \tag{12}$$

With k and h being sluggish variables while c and q are free to jump instantaneously, in order for this system to have a unique stable adjustment path (i.e., be saddle-path stable) we require that it has two negative (stable) and two positive (unstable) eigenvalues. It can be easily verified that the sign of the determinant of the matrix in (12) is positive. This, however, is consistent with there being either two negative and two positive root, four positive, or four negative roots. To ensure that we do in fact have two positive roots requires extra conditions, which unfortunately turn out to be intractable. In all of our simulations, however, we find that (12) exhibits saddlepoint behavior and we shall focus our attention on that case, as being the plausible one. However, the stable roots may quite plausibly turn out to be complex, in which case the dynamics involve cyclical behavior.[11]

Our objective is to contrast the dynamic behavior of the economy under the following three specifications of preferences: (i) conventional preferences ($\gamma = 0$), (ii) external habits ($\phi = 0$), and (iii) internal habits ($\phi = 1$). Since these scenarios differ only in terms of their demand characteristics, consumption behavior provides the crucial determinant of the differences in the adjustment processes undertaken by the economy in the three cases. Two key concepts to understanding this include the ''rate of return effect'' and the ''status effect''. These can be seen most conveniently by focussing on equation (10b) in the two cases of conventional preferences and external habits. Assuming for convenience $n = \delta = g = 0$ and letting $\hat{c} \equiv \dot{c}/c$, these become:

3.1. Conventional Preferences

$$\hat{c} = \frac{1}{\varepsilon}\{(1-\sigma)\alpha k^{-\sigma} - \beta\}. \tag{13a}$$

3.2. Outward-Looking Consumption Benchmark

$$\hat{c} = \frac{1}{\varepsilon}\left\{(1-\sigma)\alpha k^{-\sigma} - \rho\gamma(1-\varepsilon)\left(\frac{c}{h} - 1\right) - \beta\right\}. \tag{13b}$$

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11 The stability conditions in the case of the external consumption benchmark, when the dynamics reduce to a third-order system are simpler. In that case we can show that $\tilde{c} < \sigma\alpha(\tilde{k})^{1-\sigma}$, that is, the equilibrium consumption–income ratio be less than σ, is a plausible sufficient condition to ensure saddle-point stablilty. Under our assumptions, the case of conventional time-separable preferences is always saddle-point stable.

For conventional preferences, (13a), the rate of growth of consumption is determined by the difference between the real interest rate and the rate of time preference, multiplied by the intertemporal elasticity of substitution, a measure of the agent's willingness to shift consumption across periods. A high marginal product of capital will lead to a lower level of current consumption and therefore to a high rate of consumption growth; we call this the ''rate of return effect''.[12] For outward-looking agents, the reference stock is an additional variable that interacts in the determination of the consumption growth rate. For empirically plausible values of $\varepsilon\ (>1)$, if consumption is below habit, consumption in the outward-looking economy will grow slower than in the conventional economy, and vice versa. We call this the ''status effect'' and it counteracts the ''rate of return effect'', constraining the deviations in consumption from its historical level.

4. Numerical Analysis of Some Transitional Paths

To understand better the transitional dynamics we calibrate the model to reproduce some key features of actual economies. Table 1 summarizes the parameters upon which our simulations are based. Most of these are standard and non-controversial. In this regard, $\sigma = 0.65$, implying a labor share of income of 65 percent, rate of time preference $\beta = 0.04$, instantaneous intertemporal elasticity of substitution, $1/\varepsilon = 0.4$, population growth rate $n = 0.015$, and depreciation rate, $\delta = 0.05$ are well documented, while being a neoclassical model, the normalization $\alpha = 1$ is unimportant.[13]

The critical parameters pertain to the relative importance of the reference stock, γ, and the speed with which it is adjusted, ρ. We follow Carroll et al. (1997) and set $\gamma = 0.5$, $\rho = 0.2$ as benchmark values. However, information on these parameters is sparse, and we therefore conduct some sensitivity analysis based on other circumstantial evidence. Thus, based on the estimates provided by Fuhrer (2000), we also increase γ to 0.8. In addition, his results suggest a much faster speed of adjustment in the determination of the reference stock, although this estimate is obtained with a low degree of precision. A faster speed of convergence is also suggested by the application of this model to the equity premium

Table 1. Benchmark parameters.

Production parameters	$\alpha = 1$, $\sigma = 0.65$, $\delta = 0.05$, $g = 0$
Preference parameters	$\beta = 0.04$, $\varepsilon = 2.5$, $\rho = 0.2$, $\gamma = 0.5$
Population growth	$n = 0.015$

12 This effect is a combination of the ''Solow effect'', substitution effect, and the human-wealth effect. As described by Mulligan and Sala-i-Martin (1993), the ''Solow effect'' implies that, given a constant saving rate, a low level of capital will lead to a high rate of growth simply because the average product of capital is high. The other two effects tend to increased current savings (decrease current consumption) increasing investment and growth.

13 Since we are dealing with a closed economy without a government sector savings and investment coincide and we use the terms savings and investment interchangeably.

Table 2. Base equilibria.

Conventional				External and Internal Habits				
							Stable Eigenvalues	
Y/K	S/Y;	$\hat{Y}$ (%)	Stable Eigenvalues	Y/K	S/Y	$\hat{Y}$ (%)	External	Internal
0.30	0.217	1.50	$-0.0626,\ -0.20$	0.30	0.217	1.50	-0.1057 $\pm 0.0188i$ $r=0.1073$	-0.0936 $\pm 0.0290i$ $r=0.0980$

puzzle literature, and in light of this we increase ρ to 0.5, 0.8, 1.5 and 5.[14] But these adjustment speeds imply rapid convergence speeds, and from this standpoint we also consider $\rho = 0.02$ as well.

Finally, for expositional purposes it is convenient to treat the benchmark technology as one of zero productivity growth $(g = 0)$. This has the advantage that steady-state values are identical across specifications. From Table 2, we see that the base equilibrium implies an output–capital ratio of 0.3, savings rate of 21.7 percent (consumption–output ratio of 78.3 percent), and a growth rate of 1.5 percent.

4.1. Speed of Convergence

A particularly interesting aspect of the results pertains to the eigenvalues. These are crucial in determining the economy's speed of convergence, which has been the subject of both extensive empirical and theoretical analysis. The empirical evidence on convergence speeds is mixed. Early influential work by Barro and Sala-i-Martin (1992, 1995), Mankiw et al. (1992) yielded estimates of around 2–3 percent per annum, which became a benchmark estimate, although it conflicts with the predictions of the simplest neoclassical models, of around 10 percent. Subsequent work suggests that the rates of convergence are more variable, being sensitive to the time period and the set of countries, and a wider range of empirical estimates have since been obtained.[15]

Many models, including Carroll et al. (1997, 2000) have the property that the transitional dynamics are determined by a one-dimensional stable manifold. That structure imposes the restriction that all the variables converge to their respective steady states at the same constant speed equal to the magnitude of the unique stable eigenvalue. By contrast, if the stable manifold is two-dimensional (as for either of the habit formation cases) the

14 There is some difficulty in translating empirical estimates of these parameters, which are based on discrete-time models to our continuous-time formulation.

15 For example, Islam (1995) estimates the rate of convergence to be 4.7 percent for non-oil countries and 9.7 percent for OECD countries. Caselli et al. (1996), use a GMM estimator to correct for sources of inconsistency due to correlated country-specific effects and endogenous explanatory variables and obtain a convergence rate of around 10 percent. Evans (1997) using an alternative method to generate consistent estimates of convergence finds them to be around 6 percent.

speed of convergence of any variable at any point of time is a weighted average of the two stable eigenvalues. Over time, the weight of the smaller (more negative) eigenvalue declines, so that the larger of the two stable (negative) eigenvalues describes the asymptotic speed of convergence.[16] The flexibility provided by the additional eigenvalue allows the system to match some features of the data related with the timing of key variables and growth rates along the transitional path.

In a few instances the stable eigenvalues come in complex conjugate pairs, μ_1, $\mu_2 = a \pm ib$. In such a case, general solution for a variable is of the form $x(t) = \tilde{x} + Be^{-at}\cos(bt + \kappa)$, where B, κ are arbitrary constants, implying cyclical behavior with periodicity $2\pi/b$. Because the transitional path oscillates about its steady-state, the measure of convergence speed proposed by Eicher and Turnovsky (1999b) is inconvenient because it becomes infinite each time $x(t)$ cycles through its equilibrium value, $\tilde{x}$. In this case more appropriate measures of the rate of convergence are either a or the modulus of the roots, $r = \sqrt{a^2 + b^2}$, both of which are constant over time.

Turning to the base equilibrium in Table 2, the two eigenvalues under conventional time separable preferences are -0.0626 and -0.20. However, the dynamics of capital (and consumption), on the one hand, and the reference consumption level, on the other, decouple. As a result, capital and consumption converge at the constant rate of 6.26 percent, while the reference stock, which by assumption is irrelevant to the consumption–investment allocation, converges at 20 percent. In contrast, when preferences depend upon benchmark consumption, the stable dynamics of the entire system becomes interdependent, and the convergence of capital and the reference consumption level occur jointly.[17] Moreover, in this case for both specifications of preferences, the two stable roots are complex, indicating that the stable adjustment path is one of cyclical behavior. However, because the imaginary component is small, (0.019, 0.029 respectively), the periodicity of the cycles are extremely long, (330, 216 years respectively) so that for practical purposes the transitional paths are essentially non-cyclical.

Table 2 suggests that an economy with outward-looking agents converges more rapidly than does one having inward-looking agents. This is because an inward-looking agent, who takes account of the impact of his current consumption on the reference level, in effect has a lower intertemporal elasticity of substitution, thereby slowing down the rate of convergence. To provide some intuition, consider an economy that begins with the k/h ratio below its equilibrium level. In this case, the adjustment requires additional capital accumulation and/or a reduction in the reference consumption stock. An inward-looking agent considers not only the effect of a reduction in consumption on the rate of capital accumulation, but also its dampening effect on habits. But an outward-looking agent ignores the effect of his action on the future evolution of the reference stock and therefore reduces consumption below its optimal level. Under-consumption when the stable growth path requires capital accumulation accelerates the convergence process.

The interesting, and perhaps counter-intuitive observation, is that the introduction of a reference consumption stock, which one can view as a source of sluggishness, actually

16 See Eicher and Turnovsky (1999a) for further discussion of this point.

17 In early terminology of dynamic systems, the system would be said to be ''indecomposable''.

Table 3. Eigenvalues and modulus for complex roots.

ρ \ γ	0.2		0.5		0.8	
	External	Internal	External	Internal	External	Internal
0.02	− 0.0206	− 0.0207	− 0.0216	− 0.0221	− 0.0228	− 0.0244
	− 0.0603	− 0.0582	− 0.0569	− 0.0512	− 0.0534	− 0.0432
0.2	− 0.0710	− 0.0708	− 0.1057	− 0.0936	− 0.0910	− 0.0694
			$\pm 0.0188i$	$\pm 0.0290i$	$\pm 0.0510i$	$\pm 0.0486i$
	− 0.1708	− 0.1619	$r = 0.1073$	$r = 0.0980$	$r = 0.1043$	$r = 0.0847$
0.5	− 0.0686	− 0.0682	− 0.0820	− 0.0809	− 0.1109	− 0.1231
						$\pm 0.0326i$
	− 0.4381	− 0.4180	− 0.3422	− 0.2890	− 0.2331	$r = 0.1273$
0.8	− 0.0682	− 0.0678	− 0.0801	− 0.0790	− 0.1010	− 0.1027
	− 0.7028	− 0.6707	− 0.5552	− 0.4707	− 0.4016	− 0.2548
1.5	− 0.0680	− 0.0678	− 0.0789	− 0.0781	− 0.0965	− 0.0953
	− 1.3194	− 1.2587	− 1.0475	− 0.8877	− 0.7726	− 0.4841
5	− 0.0678	− 0.0677	− 0.0781	− 0.0778	− 0.0938	− 0.0931
	− 4.3998	− 4.1960	− 3.4992	− 2.9607	− 2.5978	− 1.6211

speeds up the dynamics. The speed of adjustment, -0.106, implicit in the real part of the two complex roots in the external case is essentially some kind of average of the two eigenvalues -0.0626 and -0.20 of the conventional system. Intuitively, the interaction of the capital dynamics with the more rapid dynamics of benchmark consumption in the indecomposable economy means that the convergence speed of the former is increased, while that of the latter slows down.

Table 3 performs some sensitivity analysis with regard to the eigenvalues across a wide range of values of γ and ρ. Overall, we see that the asymptotic speeds of convergence span the range 2 to 12 percent. Asymptotic speeds of convergence of around 2 percent can be mimicked only by taking implausibly low values of ρ of around 0.02. But the model can easily replicate convergence speeds of up to 10 percent, consistent with the more recent empirical evidence, for wide ranges of γ and ρ.[18]

In general, the relation between ρ and the speed of convergence of the real variables is non-monotonic.[19] The asymptotic speed of convergence of the overall system increases with ρ for low values of ρ, and decreases with ρ for high values of ρ. Taking $\rho = 0.02$, for conventional preferences, capital converges at 6.26 percent, while reference consumption now converges at only 2 percent. With external preferences and $\gamma = 0.5$, the eigenvalues are now both real (-0.0569, -0.022), so that asymptotically, the entire system—capital,

18 We may note that even though the AK model always reduces the convergence speed (making it finite), for the comparable parameterization to our benchmark, it implies a faster speed of asymptotic convergence than is obtained here (14.5 percent vs. around 10 percent).

19 For some extreme values, Table 3 implies that the asymptotic speed of convergence of the inward-looking economy slightly exceeds that of the outward-looking economy. In these cases, the other eigenvalue is substantially larger (in magnitude) to suggest that the outward-looking economy will in fact still converge more rapidly for long periods of time.

Table 4. Increase in g from 0 to 2 percent.

A. Per capita quantities

	Impact				After 50 Years			
	Cap. %Δ	Cons. %Δ	Output %Δ	Sav. %Δ	Cap. %Δ	Cons. %Δ	Output %Δ	Sav. %Δ
Conventional	0	14.3	0	− 51.87	50.7	128.2	121.1	95.1
External habits	0	9.0	0	− 32.46	74.6	134.1	132.8	128.1
Internal habits	0	8.1	0	− 29.29	74.4	134.0	132.7	128.1

B. Growth rates and ratios (percentage point change)

	Impact				Steady State			
	$\hat{K}$	$\hat{Y}$	(c/h)	(s/y)	$\hat{K}$	$\hat{Y}$	(c/h)	(s/y)
Conventional	− 1.88	0.62	—	− 11.3	2.0	2.0	—	− 2.5
External habits	− 1.29	0.83	9.0	− 7.0	2.0	2.0	10.0	− 4.0
Internal habits	− 1.10	0.90	8.1	− 6.4	2.0	2.0	10.0	− 4.0

C. Welfare evaluation

	Impact %Δ	After 50 years %Δ	Intertemporal %Δ
Conventional	14.34	128.28	39.42
External habits	18.76	157.31	53.26
Internal habits	16.86	157.20	53.31

output, and benchmark consumption—converge at 2.2 percent. In this case, the slow evolution of benchmark consumption slows everything down. At the other extreme, as $\rho \to \infty$, so that $h \to c$, the time non-separable utility model converges to the standard model, although with a higher IES. As a consequence, the convergence speed again converges to that of the standard model, although adjusted now for the higher IES.

We now focus on the dynamic response to two shocks: (i) a 2 percent increase in the growth rate of labor productivity, (ii) a 10 percent destruction in capital. These shocks differ in interesting ways. The first is non-stationary, in the sense of generating a permanent increase in the equilibrium growth rate so that per capita quantities grow indefinitely. The second leaves the steady-state growth rate unchanged at zero, so that per capita quantities converge to some stationary level.

4.2. Increase in the Growth Rate of Productivity of Labor

We begin by considering the evolution of the three economies in response to a permanent increase in the growth rate of labor productivity from 0 to 2 percent. The three panels of Table 4 summarize the short-run and long-run effects of the change on key economic variables for the three specifications of preferences. Because of the increase in the growth rate, per capita quantities grow indefinitely, converging to a balanced growth path rather than to a steady-state level. Moreover, the steady-state values of the stationary variables (defined per effective unit of labor), while convenient for deriving the formal solution, are of little economic interest *per se*. Consequently, Panel A in Table 4 describes the effect of the 2 percent increase in productivity growth on the per capita levels of consumption, capital, output, and savings, both initially immediately following the shock, and after 50 years. Panel B summarizes the effects on growth rates and ratios, both of which are stationary.

The third panel summarizes the effects of the shocks on two measures of welfare. The change in intertemporal welfare, reported in the final column, measures the change in the representative agent's optimized utility function Ω (given in (1)), when C_i, H_i are evaluated along the equilibrium path. The welfare gains reported are equivalent variation measures, calculated as the percentage change in the permanent flow of consumption necessary to equate the initial level of welfare to what they would be following the structural changes. Details of this calculation are provided in the Appendix. As long as the equilibrium growth rate, g, is not too large (as in our simulations) the intertemporal welfare change, as measured by discounted utility is finite.

In addition, we report the effects on welfare at different instants of time along the equilibrium growth path. In the Appendix we show that converting the utility measures into equivalent permanent changes in consumption flows, we can conveniently express the change in instantaneous utility at time t from the corresponding base pre-shock level (denoted by the subscript b) in terms of

$$\xi - 1 = \left(\frac{c(t)}{c_b(t)}\right)\left(\frac{(c(t)/h(t))}{(c_b(t)/h_b(t))}\right)^{\gamma/(1-\gamma)} \mathrm{e}^{(g_2 - g_1)t} - 1, \tag{14}$$

where g_1, g_2 are the equilibrium growth rates before and after the structural change. Note that if $g_1 \neq g_2$, the pre-shock and post-shock paths of instantaneous utility will diverge, ultimately growing at different asymptotic rates. Accordingly, the consumption level necessary to compensate for the change grows over time to reflect the growth differential. In the case of conventional preferences, $\gamma = 0$, the time path for instantaneous utility simply mirrors that of instantaneous consumption. However, when utility depends in part upon benchmark consumption, the percentage change in instantaneous utility comprises two components; the proportionate change in absolute consumption, together with the proportionate change in consumption relative to its benchmark level.

We proceed as follows. We first describe the responses under conventional specification of preferences when only the ''rate of return effect'' is present, and then highlight the differences resulting from the introduction of time non-separable preferences and its associated ''status effect''.

From Table 4, we see that a permanent increase in the growth rate of labor productivity of 2 percent raises the steady-state growth rates of per capita capital stock, output (and consumption) by 2 percent. After 50 years these higher growth rates accumulate to increases in the per capita stocks of capital, output, consumption, and savings of 50.7, 121, 128, and 95 percent, respectively, over what they would have been in the absence of the enhanced productivity growth. The representative agent, being forward-looking, anticipates this future increase in growth in productive capacity, and immediately increases his current consumption level by 14.3 percent. With the capital stock being fixed instantaneously, and the growth in productivity occurring over time, output remains unchanged in the short-run. Thus, the immediate increase in consumption comes at the expense of an initial dramatic decline in the savings rate from 21.7 to 10.4 percent, leading to an initial decline in the per capita growth rate of capital of 1.9 percentage points. However, this is more than offset by the enhanced productivity growth rate so that the growth rate of per capita output initially increases by 0.6 percentage points. Over time, the accumulated effects of the higher productivity growth rate generates sufficient additional output so that eventually there are increases in both savings and consumption, despite the slight long-run decline in the saving rate.

Figure 1 illustrates the adjustment of key variables in the economy. Figure 1(a) shows the initial decline in the per capita capital stock, which after around 15 years has fallen to around 90 percent of its pre-shock level. Thereafter, the accumulated increase in output resulting from the higher productivity growth permits growth in both the per capita stock of capital and consumption to occur. After around 25 years the per capita stock of capital recovers its pre-shock level, and it will then increase steadily thereafter. Per capita output growth, implied by Figure 1(d) increases steadily from 0.6 to 2 percent. The time paths for consumption and savings are illustrated in Figure 1(b), (c), and (e).

Figure 1 also highlights how the time paths for both inward-looking and outward-looking economies track each other closely, although they are quite distinct from those of the conventional case, a fact that is also reflected in Table 4. When preferences are conditioned by the presence of a reference consumption level, the initial response is a relatively smaller increase in consumption and therefore, given initial output, a relatively smaller decline in savings. The initial increase in consumption is limited by the ''status effect'', meaning that the utility associated with any short-run increase in consumption relative to the reference stock is dampened, thereby reducing the incentive to consume. In the case of inward-looking agents, the initial increase in the consumption level is reduced to 8.1 percent, thus allowing a 32.5 percent decrease in savings, relative to its initial level, and a decrease in the savings rate from 21.7 to 15.3 percent. This leads to a substantially smaller initial reduction in the growth of capital, relative to the conventional case, as seen in Figure 1(a). This is reflected in the growth rate of output in Figure 1(d), which now increases by 0.9 percentage points. After around 10 years, the capital stock in the inward-looking economy will exceed that of the conventional economy by a sufficient amount so that its consumption level will begin to overtake it as well, as seen in Figure 1(b).

The case of an externally generated reference consumption level operates in much the same way, though with one difference. This is because the agent now ignores the fact that current consumption also contributes positively to the evolution of his reference consumption stock reducing his future satisfaction. As a result, the transition in this case is characterized by initial over-consumption relative to the inward-looking economy, followed by subsequent under-consumption, during later phases of the transition (see Figure 1(b)). However, these differences are very small, the initial increase in consumption being 9 rather than 8.1 percent, and declining over time.

A striking feature of the dynamics is the non-monotonic adjustment in the ratio c/h, which follow similar paths for both types of reference points. Starting from the steady-state with $c/h = 1$, the initial jump in c with h sluggish leads to an immediate increase in this ratio to 1.09. With $\rho = 0.20$, h begins to adjust at a faster rate than does c, so that c/h

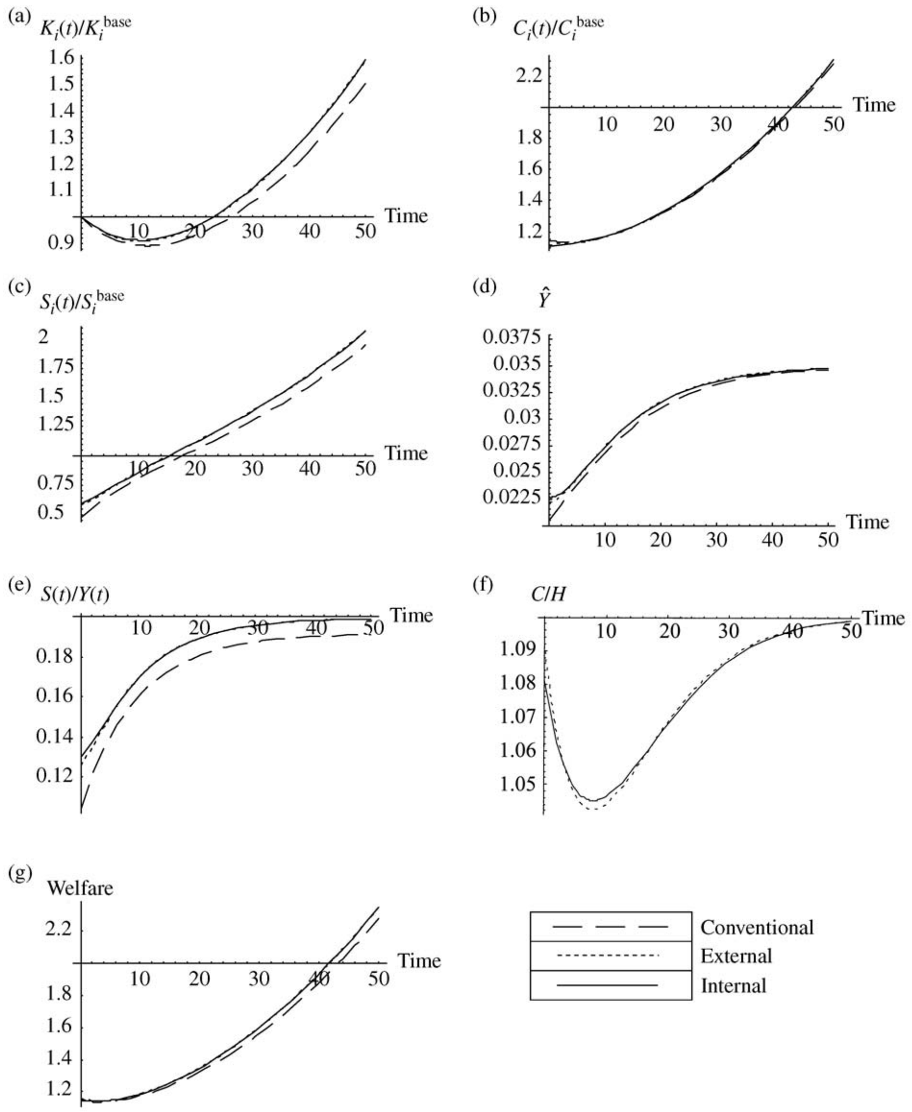

Figure 1. Transitional dynamics after an increase in the rate of technological change, g, from 0 to 2 percent. (a) Time paths for per capita capital stock relative to base. (b) Time paths for per capita consumption relative to base. (c) Time paths for per capita savings relative to base. (d) Growth rate of output. (e) Time paths for savings rate. (f) Time paths for ratio of actual to reference cons. (g) Time paths for instantaneous welfare.

begins to decline. After about 8 years of increased technological growth, output and consumption begin to increase at a faster rate, so that c/h begins to increase, eventually increasing to 10 percent above its original value.

The time paths for instantaneous welfare corresponding to the alternative specifications for preferences are illustrated in Figure 1(g). We immediately see from (14) that the initial

14.3 percent increase in consumption under conventional preferences $(\gamma = 0)$ leads to a corresponding initial 14.3 percent increase in welfare. Both forms of time non-separable preferences lead to a smaller increase in initial consumption of only around 9 percent, which raises utility by the same percentage amount. But in addition, with habits being sluggish (and fixed instantaneously) this raises short-run relative consumption by around 9 percent, so that overall, welfare increases by around 18 percent in the short run.[20] Over time, as habits begin to catch up to current consumption, the relative consumption term in (14) declines to 1.1 and asymptotically, welfare changes all converge to the common long-run growth rate of 2 percent. After 50 years, the welfare levels for both external and internal references are virtually identical, both being around 157 percent higher than they would have been without the higher productivity growth rate, and significantly higher gains than with conventional preferences. The different intertemporal measures of welfare reflect the differences along the transitional time paths. Again both time non-separable preference functions suggest similar welfare gains of around 53.3 percent, also significantly higher than the 39.4 percent implied by conventional preferences.

These results have two interesting implications. First, despite the fact that agents having time non-separable preferences enjoy smaller short-run absolute consumption gains than those having conventional preferences, in response to the growth in productivity, they nevertheless enjoy larger short-run utility gains.[21] This is because they also derive benefits from the relative change involved. Second, if preferences are in fact time non-separable as much recent empirical evidence suggests, the welfare conclusions obtained under the assumption of time separability could be highly misleading, substantially understating the true benefits to the agents.

4.3. Destruction of Capital

Table 5 summarizes the effects of a temporary 10 percent destruction of capital, brought about by a war or natural disaster such as an earthquake. Figure 2 illustrates the dynamics in the benchmark case of zero productivity growth. This shock is stationary, in the sense that following the shock, the economy ultimately returns to its initial growth rate. We see that for all three specifications of preferences this leads to an initial reduction in output of 3.6 percent. In the case of conventional preferences, this causes an initial reduction in consumption of around 4.4 percent and savings of around 1 percent, leading to an initial increase in the savings rate of around 0.6 percentage points to around 22.3 percent, and to a gradual restoration of the capital stock at its original level. The initial decline in consumption leads to an equivalent initial reduction in welfare of around 4.4 percent.

20 With external habits leading to slightly more consumption in the short-run, they are therefore associated with slightly higher short-run welfare.

21 This comparison needs to be interpreted with care. We do not mean to compare the welfare of agents having time-separable utility functions with those of agents having time-dependent utility functions. Instead, we are suggesting that an analysis based on time-separable utility would understate the short-run benefits derived by an agent having time-dependent utility.

Table 5. 10 percent destruction of capital.

A. Impact effects

	Per-capita Quantities				Growth Rates and Ratios			
	Cap. %Δ	Cons. %Δ	Output %Δ	Sav. %Δ	$\hat{K}$ %ptΔ	$\hat{Y}$ %ptΔ	(c/h) %ptΔ	(s/y) %ptΔ
Conventional	− 10	− 4.36	− 3.62	− 0.93	0.67	0.24	—	0.6
External habits	− 10	− 3.83	− 3.62	− 2.88	0.56	0.20	− 3.8	0.1
Internal habits	− 10	− 3.32	− 3.62	− 4.72	0.43	0.15	− 3.3	− 0.3

B. Welfare effects

	Impact %Δ	Intertemporal %Δ
Conventional	− 4.36	− 1.72
External habits	− 7.50	− 2.02
Internal habits	− 6.52	− 2.01

C. Sensitivity analysis: effect on short-run savings ratio percentage point change

		$\gamma = 0.2$	$\gamma = 0.5$	$\gamma = 0.8$
	Conventional	0.6	0.6	0.6
$\rho = 0.2$	External habits	0.4	0.1	− 0.1
	Internal habits	0.2	− 0.3	− 0.9
$\rho = 0.5$	External habits	0.4	0	− 0.3
	Internal habits	0.2	− 0.4	− 1.2
$\rho = 0.8$	External habits	0.3	0	− 0.4
	Internal habits	0.2	− 0.5	− 1.3
$\rho = 1.5$	External habits	0.3	− 0.1	− 0.5
	Internal habits	0.2	− 0.5	− 1.4
$\rho = 5$	External habits	0.3	− 0.1	− 0.6
	Internal habits	0.2	− 0.5	− 1.5

However, the monotonic increase in consumption back to its pre-shock level reduces the present value of the overall welfare loss throughout the transition to approximately 1.7 percent.

The introduction of an internally generated reference stock leads to substantial differences in the adjustment paths following a temporary destruction of capital. The initial reduction in output is again around 3.6 percent. This time, the existence of the reference stock inhibits the decline in initial consumption, which now falls by only 3.3 percent, leading to an immediate decrease in the savings rate of 0.3 percentage points, reducing it to 21.4 percent. In the short run, following the initial destruction, the ‘‘status effect’’ limits the capacity of consumption to adjust. Therefore, savings increases faster than does output so that the savings rate begins to rise. After about 12 years, the growth

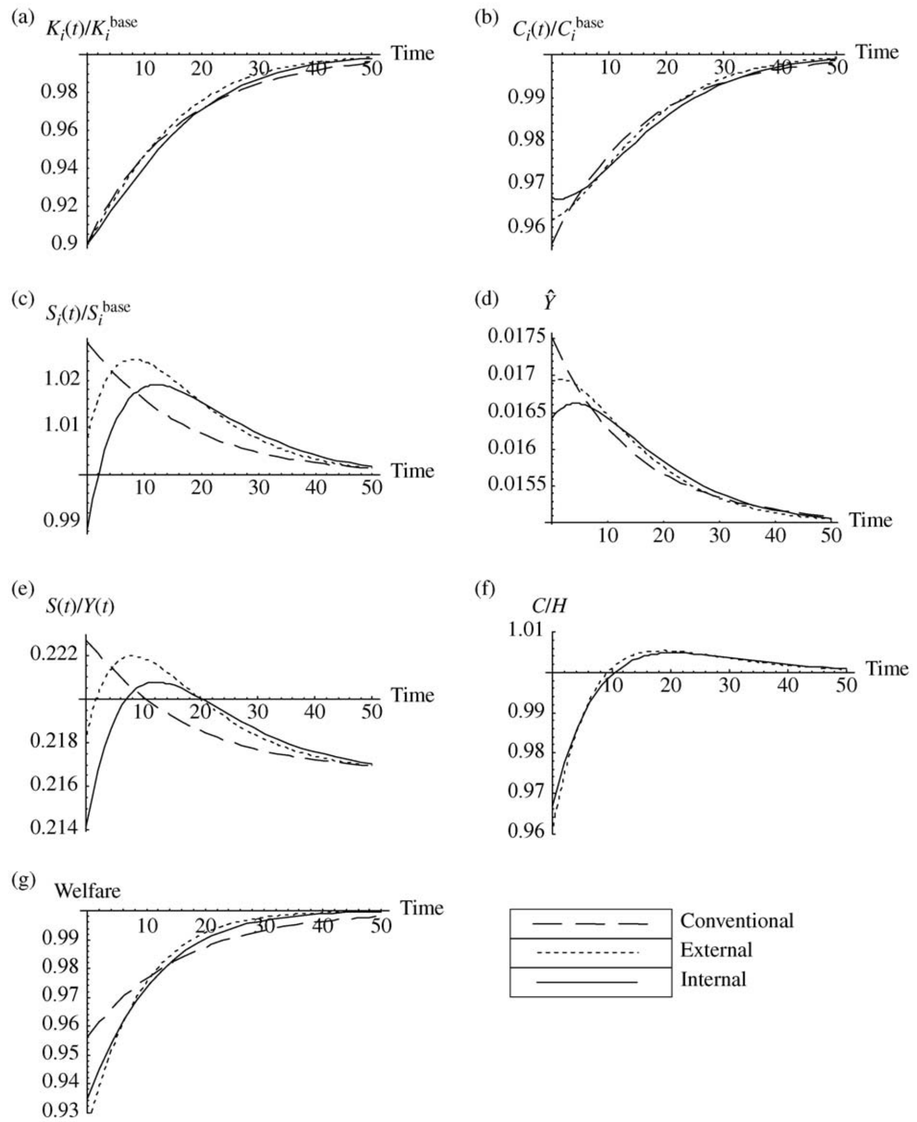

Figure 2. Transitional dynamics after a 10 percent destruction of capital. (a) Time paths for per capita capital stock relative to base. (b) Time paths for per capita consumption relative to base. (c) Time paths for per capita savings relative to base. (d) Growth rate of output. (e) Time paths for savings rate. (f) Time paths for ratio of actual to reference cons. (g) Time paths for instantaneous welfare.

rate of savings catches up to that of output and the savings rate peaks at around 22.1 percent. Thereafter, as the effects of the past increases in consumption are incorporated into the reference stock, current consumption increases and the savings rate declines, doing so monotonically until the new equilibrium is reached.

When the consumption reference stock is externally generated, agents ignore the fact that a reduction in current consumption lowers the future reference stock, and this leads to a transition characterized by under-consumption. The initially larger reduction in consumption allows for an immediate increase in the saving rate of 0.1 percentage point, thereafter the adjustment path is qualitatively similar to the inward-looking case.

The contrast in the behavior of the savings rate between the time non-separable and conventional preferences is striking and is a reflection of the fact that that the ''status effect'' initially dominates the ''rate of return effect'' for our chosen benchmark parameters. Table 5 examines the sensitivity of this finding to the two key preference parameters, ρ and γ. The results suggest that while the result is relatively insensitive to ρ, it is much more sensitive to γ. Indeed if the weight on habits is reduced to $\gamma = 0.2$, then the rate of return effect dominates and all three specifications of preferences lead to an initial increase in the savings rate.

The time paths for instantaneous welfare corresponding to the alternative specifications for preferences are illustrated in Figure 2(g). Using (14), we see that for conventional preferences, $\gamma = 0$, the time path for instantaneous utility simply mirrors that of instantaneous consumption, declining by around 4 percent initially. For the assumed value of $\gamma = 0.5$, the initial decline in welfare for both internal and external shocks reflects the decline in absolute and relative consumption. Thus welfare in the two cases immediately declines by around 6.5 and 7.5 percent respectively. But with capital adjusting more rapidly with internal habits and most rapidly with external habits the initial decline in welfare is eliminated more rapidly as we move from the conventional, to the internal, to the external habits cases. After about 15 years, the initial welfare ranking will be reversed. Despite the rather different time profile of welfare costs, these are more or less offsetting, so that the intertemporal welfare losses of the destruction of capital with either inward-looking or outward-looking agents is around 2 percent, slightly higher than the 1.7 percent for the conventional utility function.

One interesting contrast from the shock to productivity growth is that there is a greater divergence between the two specifications of time non-separable preferences, particularly during the early phases of the adjustments. This is most clearly evident in the behavior of the savings rate, $s(t)/y(t)$, which begin to converge only after around 20 years.

The analysis of this shock provides some interesting insight into the relationship between growth and savings. Empirical evidence summarized by Carroll et al. (2000) suggests that growth leads to savings. But conventional growth models have the property that growth and savings are contemporaneously related and cannot capture adequately this type of Granger–causal relationship. In contrast, comparing Figure 2(d) and (e) in the two cases with time-separable preferences, we see that the initial increase in the growth rate from 1.5 to around 1.7 percent precedes the increase in the savings rate by several periods; i.e., growth leads savings, consistent with the empirical evidence.[22]

Intuitively, the initial destruction in the capital stock raises its marginal product. Since

22 Carroll et al. (2000) also address the relationship between savings and growth in their model based on the AK technology. Since this model generates only monotonic dynamics, they are unable to discuss the timing of the causality that our higher order dynamic model with its non-monotonic dynamics enables us

the presence of a benchmark consumption level limits the initial reduction in consumption, the initial increase in the growth rate, following the capital loss, is slightly less than it would be if utility were time non-separable. As agents become ''habituated'' to low levels of consumption, a large share of the additional output is saved, leading to a progressive increase in the savings rate. As capital recovers, the marginal product of capital declines, reducing the growth rate of output and the savings rate, both of which now converge to their respective equilibrium. The non-monotonic paths of both these variables is due to the ''status effect'', which initially inhibits the decrease in consumption, together with the ''rate of return effect'', which slows down the eventual decline in the growth rate.

5. Some Sensitivity Analysis

Our analysis has introduced two critical parameters: (i) the speed of adjustment of the reference stock, ρ and (ii) the weight of habit in preferences, γ. We have conducted an extensive sensitivity analysis with respect to both parameters, allowing ρ and γ to vary from 0.2 to 5, and 0.2 to 0.8, respectively (as in Table 5). The overall conclusion is that our results are generally robust with respect to plausible parameter changes and here we report only the main results.[23]

5.1. Speed of Adjustment of Reference Stock

Since the time-separable preferences are independent of both ρ and γ, we need focus only on how the two types of habit formation change. We may note that as $\rho \to 0$ or $\rho \to \infty$ the paths of the time non-separable economies converge to the conventional case.

We consider first the increase in the productivity growth rate from 0 to 2 percent. The similar behavior exhibited by both inward-looking and outward-looking economies for the benchmark economy continues to prevail, and accordingly, we shall focus our attention on the former. Increasing ρ from 0.2 to 0.8 reduces the initial increase in consumption from 8.1 to 6.6 percent, so that the savings rate initially declines from 21.7 to 16.2 percent rather than to 15.3 percent. Lower short-run consumption reduces welfare in the short-run (relative to the benchmark), as greater resources are devoted to capital accumulation, and indeed, on impact, welfare increases by only 13.7 percent, rather than 16.9 percent. In the short-term, this increases the growth rate of output and the savings rate continues to fall. At the same time, the initial more rapid rate of adjustment of habits reduces the consumption–habits ratio, more than offsetting the positive effects of more consumption, so that after the initial increase, welfare falls during the first two periods.

to do. Instead, they evaluate the derivative of the gross savings rate with respect to the growth rate of output (following an increase in productivity), showing that habits are likely to make the relationship between savings and growth more positive.

23 More detailed summary tables of our sensitivity analysis and illustrative graphs are available from the authors.

However, over time, the increase in absolute consumption more than offsets the decline in relative consumption and welfare begins to rise. As ρ increases further, the decline in welfare during the early phase becomes more pronounced.

The time paths for per capita capital stock, consumption, output, and the savings ratio are all generally insensitive to changes in ρ. But since a larger ρ implies that habits are adjusting to current consumption at a faster rate, the relative consumption effects of welfare decline (cf. (14)) with ρ. As a result, the overall welfare gains resulting from the productivity gain decline correspondingly. These differences become substantial as γ increases, and the role of habits increases in importance. Thus, for example, if $\gamma = 0.2$, the overall intertemporal welfare gains decline from 43.1 to 40.9 percent as ρ increases from 0.2 to 5, whereas if $\gamma = 0.8$, the corresponding decline is from 92.9 to 50.2 percent.

A similar sensitivity analysis in the case of a 10 percent destruction of capital has also been conducted. The most striking feature is that increasing the speed of adjustment of the reference stock causes consumption to continue to decline following the initial shock, in the case of time non-separable preferences, doing so for about two periods. This has two effects on the ''hump'' in the savings ratio; it both accentuates it and pushes it forward in time, so that if $\rho = 1.5$ it peaks after about 2–3 periods, rather than in about 10–12, as in the benchmark case.

5.2. Weight of Habits in Preferences

As γ declines toward zero, the contribution of relative consumption declines and the time non-separable specification converges to the conventional time separable case. As γ increases, the time non-separable economies respond to an increase in the productivity growth rate with smaller initial increases in consumption. Intuitively, as the weight of relative consumption increases, smaller increases in the level of consumption are enough to achieve larger increases in instantaneous welfare. The possibility of the time non-separable economies to substitute relative consumption for absolute consumption allows them to achieve larger increases in savings, capital accumulation and growth. If $\gamma = 0.8$, the weight assigned to relative consumption is so large that welfare initially increases by around 33.3 percent for internal habits, and over time, instantaneous welfare follows a path similar to Figure 1(g).

In the case of the destruction of capital, an increase in the weight of relative consumption reduces the initial decline in consumption following the shock, in the case of time non-separable preferences. At the same time, consumption continues to decline for several periods thereafter, particularly for the inward-looking economy. The combination of these two effects tends to postpone but accentuate the ''hump'' in the savings ratio that this shock generates.

6. Time Non-Separable Preferences: AK vs. Neoclassical Technology

As we noted at the outset, Carroll et al. (1997, 2000) examine the dynamics of the basic Rebelo (1991) endogenous growth model under time non-separable preferences. The introduction of a second state variable, benchmark consumption, introduces transitional dynamics, so that in contrast to the conventional AK model, the economy is no longer always on its balanced growth path, but now exhibits transitional dynamics. Nonetheless, the strong knife-edge conditions required to generate ongoing growth severely restrict the equilibrium dynamic behavior, essentially restricting it to monotonic adjustments exclusively driven by preference parameters.

In this section we briefly compare the results of that model, with its assumed constant return to capital (and stationary population), with those of the present model and its more flexible production technology. Carroll et al. (1997, 2000) have considered virtually the same two shocks as have we. The first is an increase in productivity, A, so as to raise the equilibrium growth rate from 1 to 2 percent, the second being the 10 percent destruction of capital. To maximize comparability, we set $n=0$, $\beta=0.05$, while assuming an initial labor productivity growth rate of 1 percent, thereby generating the same initial equilibrium growth rate of 1 percent. We then introduce a 1 percent increase in labor productivity growth, thus increasing the equilibrium growth rate to 2 percent. We also follow them by setting $\varepsilon=2$, but all other parameters remain unchanged. Since the behavior of each production technology—AK and neoclassical—is qualitatively similar across preference specifications we restrict our comparison to the ''catching up with the Joneses'' case. Figures 3 and 4 compare the transitional adjustment paths in the two economies for both types of shocks. The differences are striking in both cases, the growth rates, for example, evolving in contrasting ways along the transitional paths.

The differences in response to the productivity shock reflect the fact that they do not share precisely the same time profile, despite the fact that they both raise the long-run growth rate by 1 percent. In our model, with the declining productivity of capital, we require a sustained increase in the growth rate of productivity, in order to generate a permanently higher growth rate. The effect of this takes place only gradually over time, as we have been discussing. In contrast, for the AK technology, the increase in productivity, A, occurs fully on impact, and indeed productivity cannot grow over time for a balanced growth equilibrium to prevail. As a result, the growth rate of capital in the AK model adjusts immediately, and indeed initially over-adjusts due to the inhibiting effect of the reference consumption stock on the initial adjustment to consumption.[24]

Differences are also very pronounced for the destruction of capital shock (which is identical in the two cases). In the AK case, an economy with initial capital–habit ratio below its steady-state level will have an initial low level of consumption. The saving, capital accumulation, output, and consumption growth rates will initially be low, monotonically approaching their initial steady state levels after 30 years. But the more flexible neoclassical production technology predicts entirely different behavior. The initial

24 In a previous version of this paper we also compared the behavior of the two models in response to a one-time increase in productivity, as represented by α in this model. In this case, we find that the behavior of the two economies is qualitatively similar during the transition. The only difference is that in this model this shock has only a temporary effect on the growth rate, unlike the AK model where its effect is permanent. The key factor determining the similarity or divergence in the behavior of the two models in response to a shock is the extent to which the dynamics in the present, more flexible, model are non-monotonic.

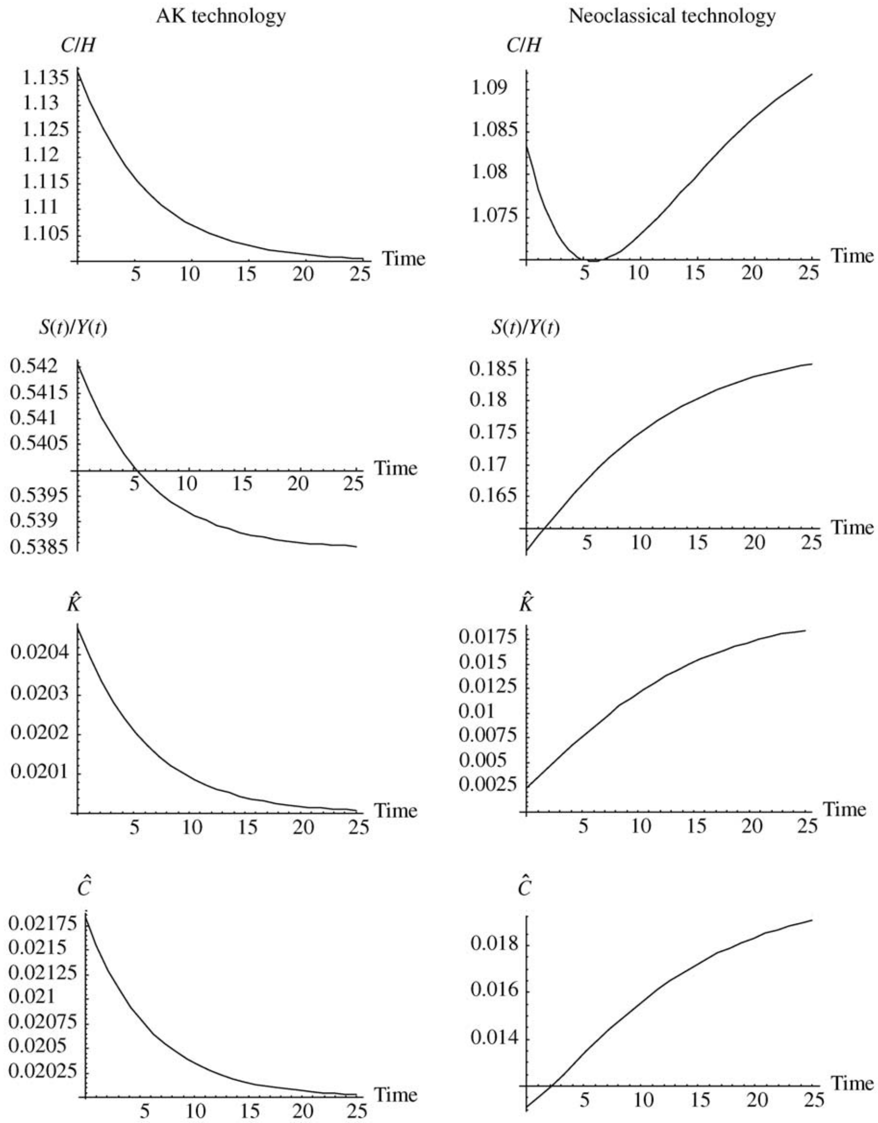

Figure 3. Transitional dynamics after an increase in g or A, respectively, that leads to an increase in equilibrium growth from 1 to 2 percent.

destruction of capital raises its marginal product leading to an immediate decrease in the level of consumption. The higher level of savings leads to an increase in the rate of capital accumulation and this higher rate of investment results in a progressive recovery of output, capital, and consumption. With a constant rate of return to capital, saving and growth remain below their steady-state levels along the transition, while if the rate of return to

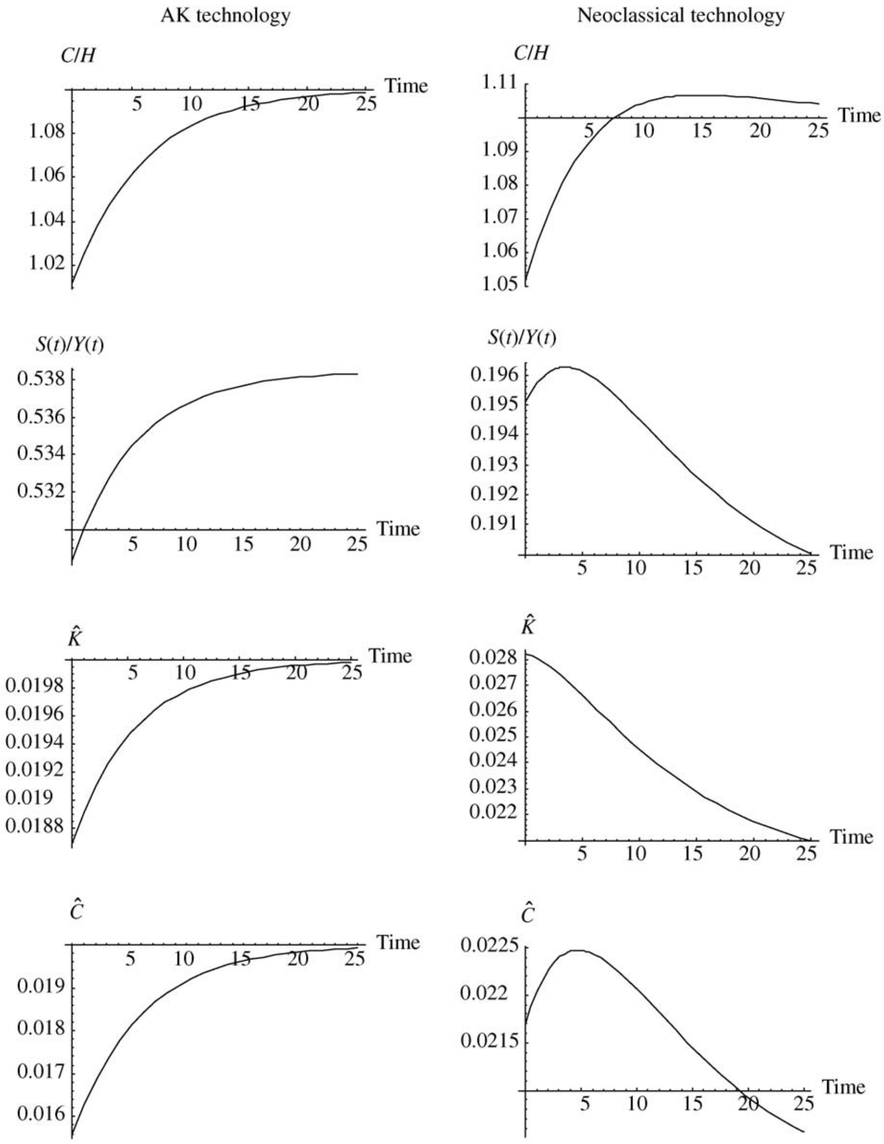

Figure 4. Transitional dynamics after a 10 percent destruction of capital under external habits.

capital is endogenously determined, saving and growth over-shoot during the transition, and approach their steady-state from above.[25]

The intuition behind these contradicting results rests on the different assumptions about the behavior of the marginal product of capital implied by each production technology. If

25 The non-montonic adjustment to this shock may also occur in the Alonso-Carrera (2001b) model.

the aggregate technology exhibits constant returns to capital, then its marginal product is independent of the level of capital, and therefore the saving–consumption decision is dominated by the predetermined reference stock. After a destruction of physical capital, an agent with a high reference stock will try to prevent consumption from falling relative to the reference level, ''the status effect'', consuming at an unsustainably high level while he allows the reference stock to adjust. This higher consumption–output ratio lowers the rates of saving, capital accumulation, and growth along the transition. On the other hand, if the aggregate technology exhibits diminishing returns to capital two counteracting effects drive the adjustment process. As for the AK technology, the ''status effect'' is present, preventing a plunge in the level of consumption. At the same time, the ''rate of return effect'' stimulates saving and capital accumulation. For this chosen parameter set the ''rate of return effect'' dominates leading to a transition characterized by above-equilibrium levels of growth and saving, although the sensitivity analysis discussed earlier suggests that this need not always be the case.

7. Conclusions

Recent empirical evidence has supported the importance of time non-separable preferences as an alternative to the conventional time separable utility function. Given the convincing nature of this evidence it is important that its consequences for the dynamics and growth of the macro economy be well understood. Previous research has focused almost entirely on the simplest AK growth model and this paper has examined the effects of introducing time non-separable preferences in the more flexible neoclassical growth framework.

This analysis has been carried out with a twofold objective in mind. First, we have compared the adjustment process of the key variables in our model, under three alternative preference specifications: (i) conventional time separable preferences, (ii) outward-looking preferences reflecting an attitude of ''catching up with the Joneses'', and (iii) inward-looking preferences that reflect habit formation. Because of the more general specifications of preferences and technology, most of our work has proceeded numerically, by calibrating a plausible macroeconomic model.

The analysis yields important differences in the behavior of consumption and saving under the two specifications of time non-separable preferences, (ii) and (iii), relative to the conventional specification, (i). These differences arise from the fact that the introduction of the reference stock ties the behavior of these variables to the past, thus limiting their ability to respond to a shock. How much (ii) and (iii) deviate from one another during a transition depends on the shock. Inward-looking and outward-looking preferences track each other quite closely in response to an exogenous increase in productivity growth, with both deviating substantially from the time path generated by conventional preferences. But for the other shock we consider, a destruction in the initial capital stock, (ii) and (iii) are less closely tied during the transition, especially in its early stages.

The second aspect we consider is to contrast the effects of time non-separable preferences under alternative production structures. We do this by comparing our results, obtained under a neoclassical production structure with those obtained under the restrictive AK technology. The most striking aspect of this comparison is the sharply contrasting responses of the two models to a 10 percent destruction of capital. Under the AK production structure, consumption, saving and the growth rate approach the new steady state monotonically from below. In contrast, our model predicts that both savings and the growth rate adjust non-monotonically, overshooting their long-run responses during the transition, consistent with empirical evidence provided by the post World War II experience of Europe and Japan. This non-monotonic behavior is possible because of the higher dimensionality of the underlying dynamic system. The intuition behind it lies in understanding the interacting forces at work. The dynamics under time separable preferences are driven by what we have called the ''rate of return effect''. On the other hand, what we have called the ''status effect'' is the engine behind the adjustment process in the time non-separable AK model. In our specification both effects play an important role during the adjustment process, combining to provide a flexible framework able to account for rich dynamic behavior.

Appendix

A.1. Elimination of Non-Optimal Equilibrium

We show how one of the solutions for capital, in the internal habit formation case violates a transversality condition and therefore can be eliminated. We first reproduce (11d), rewritten as

$$q^* = \frac{\gamma(c/h)^*}{\rho\gamma\phi(c/h)^* + n + \delta - \rho - \alpha(1-\sigma)(k^*)^{-\sigma}}. \tag{A.1}$$

Setting $\dot{c} = 0$ in (10b) and multiplying the resulting equation by $(1 - \rho q^*)$ we get

$$\alpha(1-\sigma)(k^*)^{-\sigma} + \rho\phi q^*(\beta + \rho) - (\beta + n + \delta) + z(1 - \rho q^*) = 0, \tag{A.2}$$

where $z \equiv \rho\gamma(c/h)^*(\phi - 1 + \varepsilon) + \gamma\rho(1 - \varepsilon) - \varepsilon g$.

Substituting (A.1) into (A.2) leads to

$$\begin{aligned} &\alpha(1-\sigma)(k^*)^{-\sigma} + \rho\phi\frac{\gamma(c/h)^*}{\rho\gamma\phi(c/h)^* + n + \delta - \rho - \alpha(1-\sigma)(k^*)^{-\sigma}}(\beta + \rho) \\ &\quad - (\beta + n + \delta) + z\left(1 - \rho\frac{\gamma(c/h)^*}{\rho\gamma\phi(c/h)^* + n + \delta - \rho - \alpha(1-\sigma)(k^*)^{-\sigma}}\right) = 0. \end{aligned} \tag{A.3}$$

That can be reduced to the following quadratic expression in $\alpha(1-\sigma)(k^*)^{-\sigma}$.

$$\begin{aligned} &-(\alpha(1-\sigma)(k^*)^{-\sigma})^2 + (\rho\gamma\phi(c/h)^* - \rho + 2\delta + 2n - z + \beta)\alpha(1-\sigma)(k^*)^{-\sigma} \\ &\quad + \rho\gamma\phi(c/h)^*(\rho - \delta - n) + (z - \beta - \delta - n)(\delta + n - \rho) = 0. \end{aligned} \tag{A.4}$$

We can factor this equation as follows:

$$(\delta + n - \rho - \alpha(1-\sigma)(k^*)^{-\sigma})(\alpha(1-\sigma)(k^*)^{-\sigma} - (\rho\gamma\phi(c/h)^* + \delta + n + \beta - z)) = 0. \tag{A.5}$$

The two solutions to which are:

$$k_1^* = \left(\frac{-\rho + n + \delta}{\alpha(1-\sigma)}\right)^{-1/\sigma}, \tag{A.6a}$$

$$k_2^* = \left(\frac{\beta + \delta + \gamma(1-\varepsilon)g + n + \varepsilon g}{(1-\sigma)\alpha}\right)^{-1/\sigma}. \tag{A.6b}$$

We now consider the transversality condition (6d), and note that it is equivalent to

$$\hat{\lambda}_{1i} + \hat{K}_i - \beta < 0, \tag{A.7}$$

where $\hat{\lambda}_{1i}$, $\hat{K}_i$ are steady-state growth rates, and are given by:

$$\left(\frac{\dot{\lambda}_{1i}}{\lambda_{1i}}\right) \equiv \hat{\lambda}_{1i} = \beta + n + \delta - (1-\sigma)\alpha(k^*)^{-\sigma} \tag{A.8a}$$

$$\hat{K}_i = g. \tag{A.8b}$$

Substituting the first root, (A.6a), we find $\hat{\lambda}_{1i} + \hat{K}_i - \beta = \rho + g > 0$. This violates the transversality condition, and can therefore be eliminated. The second root, (A.6b), satisfies (6d) if and only if $(1-\gamma)(1-\varepsilon)g < \beta$, a sufficient condition for which is the empirically plausible condition, $\varepsilon > 1$. Imposing this latter condition, as do Alonso-Carrera et al. (2001b), we find that the optimal steady-state solution for capital in the presence of habit formation is given by (11a).

A.2. Welfare Changes as Measured by Equivalent Variations in Income Flows

We assume that the economy is initially on a balanced growth path, (indexed by b) which is growing at an initial equilibrium growth rate, $n + g_1$. Along this path,

$$C_{i,b} = c_b A = c_b A_0 \mathrm{e}^{g_1 t}; \quad H_{i,b} = h_b A = h_b A_0 \mathrm{e}^{g_1 t},$$

where c_b, h_b are the constant ratios along the initial balanced growth path and A_0 represents the initial level of technology which conditions all subsequent output and consumption levels. The corresponding level of base welfare is given by

$$\frac{1}{1-\varepsilon}\int_0^\infty \left(C_{i,b}H_{i,b}^{-\gamma}\right)^{1-\varepsilon} \mathrm{e}^{-\beta t} dt = \frac{\left(c_b h_b^{-\gamma}\right)^{1-\varepsilon}(A_0)^{(1-\gamma)(1-\varepsilon)}}{1-\varepsilon}\int_0^\infty \mathrm{e}^{[(1-\gamma)(1-\varepsilon)g_1 - \beta]t} dt. \tag{A.9}$$

Evaluating (A.9) yields

$$\frac{\left(c_b h_b^{-\gamma}\right)^{1-\varepsilon}(A_0)^{(1-\gamma)(1-\varepsilon)}}{(1-\varepsilon)[\beta - (1-\varepsilon)g_1(1-\gamma)]} \equiv W(c_b, h_b; A_0) \equiv W_b. \tag{A.10}$$

Now consider an equilibrium transitional path with steady-state growth rate $n + g_2$. Along such a path

$$C_i(t) = c(t)A(t) = c(t)A_0 e^{g_2 t}; \quad H_i(t) = h(t)A(t) = h(t)A_0 e^{g_2 t},$$

implying an intertemporal level of welfare given by

$$\frac{1}{1-\varepsilon}\int_0^\infty \left(C_i H_i^{-\gamma}\right)^{1-\varepsilon} e^{-\beta t} dt$$
$$= \frac{(A_0)^{(1-\gamma)(1-\varepsilon)}}{1-\varepsilon}\int_0^\infty \left(c(t)h(t)^{-\gamma}\right)^{1-\varepsilon} e^{[(1-\gamma)(1-\varepsilon)g_2-\beta]t} dt \equiv W(c_a, h_a; C_0) \equiv W_a. \tag{A.11}$$

As a means of comparing these two levels of utility, we determine the percentage change in the initial level of technology, A_0, and therefore in the consumption flow over the entire base path, such that the agent is indifferent between c_b, h_b and c_a, h_a. That is, we seek to find ζ such that

$$W(c_b, h_b; \zeta A_0) = W(c_a, h_a; A_0) = W_a. \tag{A.12}$$

Performing this calculation yields

$$\frac{\left(c_b h_b^{-\gamma}\right)^{1-\varepsilon}(\zeta A_0)^{(1-\varepsilon)(1-\gamma)}}{(1-\varepsilon)[\beta-(1-\varepsilon)g_1(1-\gamma)]} \equiv \zeta^{(1-\varepsilon)(1-\gamma)} W_b = W_a,$$

and hence

$$\zeta - 1 = (W_a/W_b)^{1/(1-\varepsilon)(1-\gamma)} - 1. \tag{A.13}$$

(A.13) determines the change in the base consumption level, and thus in the consumption levels at all points of time that will enable the agent's base level of intertemporal welfare to equal that following some structural change.

The relative welfare gain at any instant of time t along the transitional path (over the base level at the corresponding time) is calculated analogously, by

$$\xi - 1 = (Z_a(t)/Z_b(t))^{1/(1-\varepsilon)(1-\gamma)} - 1, \tag{A.14}$$

where $Z_b(t) \equiv \left(c_b h_b^{-\gamma}\right)^{1-\varepsilon} e^{(1-\gamma)(1-\varepsilon)g_1 t}$, $Z_a(t) \equiv \left(c(t)h(t)^{-\gamma}\right)^{1-\varepsilon} e^{(1-\gamma)(1-\varepsilon)g_2 t}$, so that

$$\xi - 1 = \left(\left(\frac{c(t)}{c_b(t)}\right)\left(\frac{h(t)}{h_b(t)}\right)^{-\gamma}\right)^{1/(1-\gamma)} e^{(g_2-g_1)t} - 1. \tag{A.15}$$

Note that if the structural change involves a change in the steady-state growth rate the pre-shock and post-shock welfare paths will diverge, ultimately growing at different rates. Accordingly, the consumption level necessary to compensate for the change grows over time to reflect the growth differential. This is not so with intertemporal utility, as long as $(1-\gamma)(1-\varepsilon)g_2 < \beta$, as in our simulations, so that intertemporal utility remains finite.

The change in instantaneous welfare, (A.15), can also be written as

$$\xi - 1 = \left(\frac{c(t)}{c_b(t)}\right)\left(\frac{(c(t)/h(t))}{(c_b(t)/h_b(t))}\right)^{\gamma/(1-\gamma)} e^{(g_2 - g_1)t} - 1, \tag{A.16}$$

which brings out the fact that as long as $\gamma > 0$, welfare differentials depend upon absolute as well as relative consumption.

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