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A mixed Bentham–Rawls criterion for intergenerational equity: Theory and implications

Francisco Alvarez-Cuadrado, Ngo Van Long *

Department of Economics, McGill University, Montreal, Canada H3A 2T7

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ABSTRACT

This paper proposes a welfare criterion that balances the need for development and the concern for the least advantaged generations, and explores its implications. This criterion, called the mixed Bentham–Rawls criterion, moderates the effect of discounting, yet permits some degree of intertemporal trade-off. It is a weighted average of two terms: (a) the sum of discounted utilities and (b) the utility level of the least advantaged generation. We derive necessary conditions to characterize growth paths that satisfy our criterion, and show that in some models with familiar dynamic specifications, an optimal path exists and displays appealing characteristics.

1. Introduction

Intergenerational equity and sustainability have been a continuing source of concern and debate among philosophers, economists and scholars from many disciplines. Environmentalists worry that the present generations will not leave enough bequest of natural capital to future generations. At the other pole, Kant [19] found it disconcerting that earlier generations should carry the burdens for the benefits of later generations,[1] and Rawls [27, p. 253] was concerned that "the utilitarian doctrine may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for later ones that are far better off. (...) Even if we cannot define a precise just savings principle, we should be able to avoid this sort of extremes." On the other hand, Rawls [26, p. 291], [27, p. 257] felt that the maximin principle, which he advocated for intra-temporal allocations, would not be suitable for intergenerational allocations because it would not generate sufficient savings. The purpose of this paper is to propose a new welfare criterion that balances the need for development and the concern for the least advantaged generations, and to explore its implications. This criterion, which

* Corresponding author. Fax: +1 514 398 4938.
E-mail addresses: francisco.alvarez-cuadrado@mcgill.ca (F. Alvarez-Cuadrado), ngo.long@mcgill.ca (N. Van Long).

[1] In his 1784 essay, "Idea for a Universal History with a Cosmopolitan Purpose," Kant put forward the view that nature is concerned with seeing that man should work his way onwards to make himself worthy of life and well-being. He added: "What remains disconcerting about all this is firstly, that the earlier generations seem to perform their laborious tasks only for the sake of the later ones, so as to prepare for them a further stage from which they can raise still higher the structure intended by nature; and secondly, that only the later generations will in fact have the good fortune to inhabit the building on which a whole series of their forefathers...had worked without being able to share in the happiness they were preparing" [28, p. 44].

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we call the mixed Bentham–Rawls (MBR) criterion, moderates the effect of discounting, yet permits some degree of intertemporal trade-off.[2]

Our new criterion is developed as an attempt to overcome the various shortcomings of (a) discounted utilitarianism, (b) undiscounted utilitarianism, (c) maximin criterion, and (d) Chichilnisky's criterion, which are outlined below.

Apart from the philosophical objection raised by Sidgwick [30, p. 414] and his followers,[3] a major shortcoming of discounted utilitarianism is that it can lead to policy prescriptions that are patently unjust. For example, in the Dasgupta–Heal–Solow (D–H–S) model with a man-made capital stock and an exhaustible resource, while it is feasible to maintain a constant positive level of consumption for ever, utilitarianism with discounting would prescribe a path with vanishing consumption in the long run, *no matter how small the discount rate is* [10,31]. This consequence offends our sense of justice.

Undiscounted utilitarianism, though it treats all generations "equally" (in the sense of anonymity, as explained below), raises two major difficulties that have been a continuing source of discussion in the economic literature.[4] First, the well-known undiscounted utilitarian criteria such as Ramsey's criterion (using the "distance from Bliss" idea), or the over-taking criterion [15,33] suffer from their inability to rank all possible utility streams, i.e., they fail the "completeness" test.[5] Second, it has been shown that there does not exist a social welfare function that satisfy the axioms of strong Pareto and anonymity.[6]

An extreme form of egalitarianism has been proposed by some philosophers and economists: the maximin criterion, according to which one stream of utilities is better than another if and only if the utility level of the least advantaged person in the former is higher than that of the least advantaged person in the latter.[7] The maximin criterion has also been called the "Rawlsian criterion" even though Rawls [26,27] had expressed strong reservations about the use of maximin as a principle for intergenerational equity.[8] The fascination with the maximin criterion has spawned a stream of theoretical literature that seeks to characterize development paths that ensure a constant level of consumption, or constant utility, for all generations [2,4,10,18,31]. The insistence on constant consumption, however, can yield consequences that are unpalatable. As Rawls [27, p. 257] pointed out, the unmodified maximin principle would entail "either no saving at all or not enough saving to improve social circumstances," which is totally unacceptable to him, especially for the case of very poor countries with a low stock of capital.[9]

In her seminal work Chichilnisky [9] proposed a criterion which she describes as a "sustainable preference." Her social welfare function takes a simple and intuitively appealing form: social welfare is a weighted sum of two terms, the first being the conventional sum of discounted utilities, and the second term takes on a value which depends only on the

[2] Many authors would interpret intergenerational equity as non-decreasing consumption paths [3]. Others, however, do not see that discounting is necessarily unethical [1]. Pezzey [24] offered an insightful discussion of sustainability, optimality, and intergenerational concern.

[3] Sidgwick [30, p. 414] argued against discounting on the philosophical ground that "the time at which a man exists cannot affect the value of his happiness from a universal point of view; ... the interests of posterity must concern a Utilitarian as much as those of his contemporaries." Perhaps Ramsey [25] was the first economist to have articulated this problem in an infinite horizon framework. Like Sidgwick [30], Ramsey [25] considered it unethical to discount the utilities of future generations.

[4] Unaware of the contributions by economists, the philosopher Krister Segerberg [29, p. 226] posed the following problem in ethics, in his article titled "A Neglected Family of Aggregation Problems in Ethics:"

"Pascal believes that eternity consists of infinitely many days [and] that when his body is dead his soul will spend each following day in Heaven or Hell...Outcomes can be represented by infinite sequences $x_0x_1 \dots x_n \dots$, where each x_n is either 1 (Heaven) or 0 (Hell)....Problems arise when he wants to compare prospects containing both 1's and 0's. Particularly difficult is it to deal with with prospects containing infinitely many 1's and also infinitely many 0's."

[5] This was pointed out by Chichilnisky [9] and many others.

[6] Sidgwick's argument against discounting has led to the "equity principle à la Sidgwick," which is embodied in the form of the "anonymity" condition: a stream of utility $s = \{x, y, z, \dots\}$ should be judged as equal to a permuted stream $s^p = \{y, x, z, \dots\}$. Diamond [11] showed that if one requires a social welfare function $W(.)$ to satisfy the strict Pareto property, a weak form of anonymity and some kind of continuity, then $W(.)$ does not exist. Svensson [7] confirmed Diamond's non-existence result *even without requiring continuity*. Svensson [32], however, showed that if, instead of seeking a (real-valued) function, we merely look for the ability to rank infinite streams of utilities, then existence of a social welfare ordering (SWO) is ensured. Unfortunately, Svensson did not offer a constructive proof, hence "knowing that such an ordering exists does not necessarily provide a clue as to how it might be constructed." Basu and Mitra [6] recommended that we should consider "lowering our demands further and be willing to accept social welfare relations (SWRs) which are pre-orders that allow comparisons between only some pairs of utility stream but not others." Fleurbaey [14] proposed a way to construct orderings but "it still relies on non-constructive objects such as ultrafilters." They conjecture that "there exists no explicit description of an ordering that satisfies Pareto and indifference to finite permutations" (p. 794). Zame [34] proved that if an ethical relation which respects strong Pareto and finite anonymity exists, it cannot be explicitly described (p. 187). Some characterizations can be achieved by adding further desirable properties. Asheim and Tungodden [5] provided a characterization of the leximin principle, by adding to strong Pareto and finite anonymity two more properties: Hammond equity and strong preference continuity. Bossert et al. [8] characterized all orderings that satisfy strong Pareto, anonymity, and the strict transfer principle.

[7] In an infinite horizon framework, Lauwers [20] axiomatized the Rawlsian criterion by relaxing strong Pareto and requiring a version of Hammond equity. Asheim and Tungodden [5] characterized different versions of leximin.

[8] As Rawls [26] put it, the maximin principle "does not apply to the savings problem... The principle is inapplicable and it would seem to imply... that there be no savings at all." See [22] for a fuller discussion of Rawls's reservations, and a review of the related literature.

[9] The absence of saving is a concern for Rawls [27], especially if one is considering a society with a very low initial level of capital. Why? Because, "to establish effective just institutions within which the basic liberties can be realized," a society must have a sufficient material base. Generations must "carry their fair share of the burden of realizing and preserving a just society."

limiting behavior of the sequence of utilities.[10] The Chichilnisky social welfare function displays a number of desirable properties: (i) non-dictatorship of the present,[11] (ii) non-dictatorship of the future,[12] and (iii) strong Pareto. Unfortunately, while Chichilnisky's social welfare function is well defined and can rank all utility sequences, in many economic models of interest, there does not exist a utility stream that isoptimal under that criterion.[13] To illustrate, take the standard neoclassical growth model, and choose a time path of saving rate to maximize social welfare under the Chichilnisky criterion. Any path that approaches (in the limit) the golden rule level of capital stock, k_g, will maximize the second term of the weighted sum. It pays therefore to delay the approach to k_g as much as possible, and stay near the modified golden rule level, k_m, as long as possible, because doing so would increase the value of the first term, and would not affect the second term. It follows that among all paths that approach k_g asymptotically, any feasible path is inferior (according to the Chichilnisky social welfare function) to some other feasible path. The failure of the Chichilnisky's criterion to yield an optimal path is disheartening.

In this paper, we propose a welfare criterion that preserves the essence of Chichilnisky criterion yet at the same time yields an optimal path (unlike the Chichilnisky criterion which does not). This new welfare criterion has two major attractions: first, it is morally compelling and, second, it overcomes the failure of the Chichilnisky's criterion to yield an optimal path. Our criterion is a weighted average (with strictly positive weights) of two terms. The first term is the conventional sum of discounted utilities, and the second term is the utility level of the least-advantaged generation. We call this new criterion the mixed Bentham–Rawls criterion. We use the word "Bentham" because our criterion is utilitarian, in the sense that it permits trade-offs of utilities of different individuals, and the word "Rawls" because of its special (but not exclusive) emphasis on the well-being of the least advantaged.[14] Clearly, the positive weight on the first term implies non-dictatorship of the future; we show that the positive weight on the second term (which is the welfare of the least advantaged generation) also ensures non-dictatorship of the present.

Chichilnisky did not obtain our criterion because she postulated an extra axiom, which she called "independence," and which means that if society is willing to trade an increase, say by δ "utility units," in the utility of generation t_1 for a decrease of ε "utility units," in the utility of generation t_2, then this ratio δ/ε should remain the same regardless of the utility levels of the two generations. We do not find this "independence axiom" compelling (in fact a better name for it would be "linearity axiom"). Our MBR criterion, by giving more weight to the least advantaged generation, does not satisfy this independence axiom.

In order to compare the solution under our new criterion with the well-known Rawlsian and utilitarian solutions we present a series of numerical exercises in the context of a simple model of renewable resource extraction. Under standard parameter values we find that when the initial stock of resource is above the golden rule stock of resource, our MBR criterion chooses a path of falling consumption, reaching (in finite time) a steady-state resource stock that exceeds the utilitarian steady state (the modified golden rule stock of resource) by almost 30% and that falls short of the golden rule capital stock by about 35%. On the other hand when the initial resource stock is half of the modified golden rule stock of resource, the MBR criterion chooses an initial constant level of consumption (lasting for nine generations) that, although it amounts to only 85% of the Rawlsian consumption level, still exceeds utilitarian consumption by almost 25%, and the consumption path takes off afterwards, reaching (asymptotically) the utilitarian steady state.

2. The mixed Bentham–Rawls criterion

Consider an economy with infinitely many generations. Since we wish to focus on the question of distributive justice among generations, we make the simplifying assumption that within each generation, all individuals receive the same income and have the same tastes. Thus, by assumption, the question of equity within each generation does not arise. This framework has been used in [9,10,12,18,23,31].

Let c_t denote the vector of consumption (of various goods and services) allocated to the representative individual of generation t. Let $u_t \equiv u(c_t)$ be the life-time utility of this individual (u_t is a real number, and $u(.)$ is a real-valued function). We interpret "utility" as "standard of living" of individuals, rather than some kind of happiness they get when consuming and/or contemplating their children' and grandchildren' life prospects. To fix ideas, it is convenient to assume that each individual lives for just one period. Consider for the moment two alternative projects, denoted by 1 and 2. Project i

[10] For example, with the utility sequence $\{u_t\}$ the second term could be lim $\sup_{t\to\infty} u_t$, or lim $\inf_{t\to\infty} u_t$, or some weighted average of these two limiting values.

[11] A social welfare function $W(.)$ is said the display "dictatorship of the present" if for any two sequences of utilities, say $\mathbf{u} = \{u_t\}$ and $\mathbf{v} = \{v_t\}$, and $W(.)$ ranks $\mathbf{u}$ higher than $\mathbf{v}$, there exists some time $T>0$ such that no modification of the tail-ends (beyond T) of $\mathbf{u}$ and $\mathbf{v}$ could reverse the ranking. The conventional utilitarian criterion with discounting implies dictatorship of the present.

[12] A social welfare function $W(.)$ is said the display "dictatorship of the future" if whenever $W(.)$ ranks $\mathbf{u}$ higher than $\mathbf{v}$, all modifications of $\mathbf{u}$ and $\mathbf{v}$ that do not affect their limiting behavior would preserve the original ranking.

[13] There exists a simple model of non-renewable resource where the Chichilnisky's criterion does identify an optimal path [9,13].

[14] In the context of intergenerational equity, the emphasis by Rawls on the least advantaged is not exclusive: if the least fortunate generation is the first generation, he still wants the first generation to save (to reduce their standard of living) for the sake of the (already better off) future generations [26, p. 291].

(where $i = 1, 2$) yields an infinite stream of utilities denoted by

$$\mathbf{u}^i \equiv \{u_t^i\}_{t=0,1,2,\ldots,} \equiv \{u_0^i, u_1^i, \ldots, u_t^i, u_{t+1}^i, \ldots\}$$

where u_t^i stands for $u(c_t^i)$.

We assume that while an individual of generation t might care about the consumption vector of his/her son or daughter, c_{t+1}, and that of his/her[15] grand-son or grand-daughter, c_{t+2}, these vectors have no impact on the "utility" level u_t. This is why it might be preferable to refer to u_t as the "standard of living" rather than "utility" of generation t.

To simplify matters, we assume that the function $u(.)$ is bounded.

Assumption 1 (*Boundedness*). Utility is bounded: $0 \leqslant u(c) \leqslant B$.

Remark. The number B is the highest possible level of utility. We shall refer to B as the "bliss utility level." Without loss of generality, we choose the normalization $B = 1$.

We use the symbol $\mathbf{u}^i$ to denote the utility stream $\{u_t^i\}_{t=0,1,2,\ldots,}$. Roughly speaking, a welfare criterion is a way of ranking all possible utility streams. Let S be the set of all possible utility streams. A social welfare function, denoted by $W(.)$, is a function that maps elements of S to the real number line.[16] It must be able to rank all possible utility sequences. We call this the "completeness property."

Property 1 (*Completeness*). *A social welfare function must rank all possible utility sequences.*

We list below two examples of social welfare functions:

(i) sum of discounted utilities (or discounted utilitarianism)

$$W^d(\mathbf{u}^i) = u_0^i + \frac{u_1^i}{(1+\delta_1)} + \frac{u_2^i}{(1+\delta_1)(1+\delta_2)} + \frac{u_3^i}{(1+\delta_1)(1+\delta_2)(1+\delta_3)} + \cdots + \cdots \tag{1}$$

where $\delta_t > 0$ for all t. According to this criterion, a utility stream $\mathbf{u}^1$ is ranked higher than a utility stream $\mathbf{u}^2$ if and only if $W^d(\mathbf{u}^1) > W^d(\mathbf{u}^2)$. Thus, a small decrease in the utility level of an individual (no matter how disadvantaged he already is) can be justified by some increase in the utility level of some other individuals.

(ii) maximin:

$$W^m(\mathbf{u}^i) = \inf\{u_t^i\}_{t=0,1,2,\ldots,} \tag{2}$$

According to this criterion, a utility stream $\mathbf{u}^1$ is ranked higher than utility stream $\mathbf{u}^2$ if and only if the utility level of the worst off generation in stream $\mathbf{u}^1$ is higher than the utility level of the worst off generation in stream $\mathbf{u}^2$, that is, if and only if,

$$\inf\{u_t^1\}_{t=0,1,2,\ldots,} > \inf\{u_t^2\}_{t=0,1,2,\ldots,}$$

Maximin has been criticized by Rawls [26,27] as an inappropriate criterion for choice among intergenerational allocations. In addition, many economists (e.g. [9]) have pointed out that it is insensitive to the utilities of generations that are not the poorest. According to the maximin welfare function (2), an increase in the utility of any generation that is not the least advantaged does not raise social welfare W^m. It seems reasonable to insist that an acceptable social welfare function must satisfy the following "strong Pareto property" [8,9]:

Property 2 (*Strong Pareto*). *Social welfare is increasing in* u_t.

Under Property 2, increasing the utility level of one generation, leaving the utility of other generations unchanged, will increase the social welfare. The social welfare function (1) satisfies Property 2, but, as Chichilnisky [9] pointed out, all utilitarian criteria with discounting place too much emphasis on the present. In fact these criteria display insensitivity to the utility of distant generations. To formalize this idea, let us follow Chichilnisky and define $({}_T\mathbf{u}^i, \mathbf{a}_T^i)$ to be a utility sequence obtained from $\mathbf{u}^i$ by replacing all elements of $\mathbf{u}^i$ except the first $T+1$ elements by the tail of a utility sequence $\mathbf{a}^i$, where

$$\mathbf{a}_T^i \equiv \{a_{T+1}^i, a_{T+2}^i, \ldots\}, \quad {}_T\mathbf{u}^i \equiv \{u_0^i, u_1^i, \ldots, u_T^i\}$$

$$({}_T\mathbf{u}^i, \mathbf{a}_T^i) \equiv \{u_0^i, u_1^i, \ldots, u_T^i, a_{T+1}^i, a_{T+2}^i, \ldots\}$$

Consider the following definition:

[15] To avoid repetitive uses of his/her etc., in all that follows, when referring to hypothetical persons, we use the masculine gender, on the understanding that it embraces the feminine gender.

[16] This definition of "social welfare function" is quite common [6,7,9]. This is to be distinguished from Arrow's use of the term "social welfare function" which is a mapping from the space of all possible individual preference orderings (over all possible social states) to the space of social orderings.

Definition 1 (*Dictatorship of the present*). A social welfare function $W(.)$ is said to display "dictatorship of the present" if the following condition holds: for every pair $(\mathbf{u}^1, \mathbf{u}^2)$, $W(\mathbf{u}^1)$ is greater than $W(\mathbf{u}^2)$ if and only if for all T sufficiently large,[17] $W({}_T\mathbf{u}^1, \mathbf{a}^1_T) > W({}_T\mathbf{u}^2, \mathbf{a}^2_T)$ for all pairs of utility sequences $(\mathbf{a}^1, \mathbf{a}^2)$, where $({}_T\mathbf{u}^i, \mathbf{a}^i_T)$ means that all elements of $\mathbf{u}^i$ except the first $T+1$ elements are replaced by the tail of the sequence $\mathbf{a}^i$.

In other words, dictatorship of the present means that any modification of utility levels of generations far away in the future would not be able to reverse the welfare ranking of a pair of utility streams. The utilitarian criterion with positive discounting displays dictatorship of the present.

A social welfare function is said to display "non-dictatorship of the present" if, regardless of how large T is, there exists some pair $(\mathbf{u}^1, \mathbf{u}^2)$ such that (a) $W(\mathbf{u}^1) > W(\mathbf{u}^2)$, and (b) one can find some modifications $(\mathbf{a}^1_T, \mathbf{a}^2_T)$ such that the ranking is reversed, i.e., $W({}_T\mathbf{u}^1, \mathbf{a}^1_T) < W({}_T\mathbf{u}^2, \mathbf{a}^2_T)$.

Let us turn to the other extreme, and consider some welfare criteria that pay no attention to the utility levels of generations that are living at the present or in the "near future." Given a utility sequence $\mathbf{u} = \{u_t\}_{t=1,2,\ldots,} \equiv \{u_0, u_1, \ldots, u_t, u_{t+1}, \ldots, \ldots, \ldots, \ldots\}$, let us consider the tail beginning at t, $\{u_t, u_{t+1}, \ldots\}$, and define the number z_t and y_t to be, respectively, the greatest lower bound and the least upper bound of this tail

$$z_t \equiv \inf\{u_t, u_{t+1}, \ldots\}$$

$$y_t \equiv \sup\{u_t, u_{t+1}, \ldots\}$$

The resulting sequence $\{z_t, z_{t+1}, \ldots\}$ is by construction of a non-decreasing sequence, and hence must converge to a limit $\bar{z}$: $\lim_{t\to\infty} z_t = \bar{z}$, i.e.,

$$\lim_{t\to\infty} \inf\{u_t, u_{t+1}, \ldots\} = \bar{z}$$

Similarly, the sequence $\{y_t, y_{t+1}, \ldots\}$ is by construction of a non-increasing sequence, and hence must converge to a limit $\bar{y}$: $\lim_{t\to\infty} y_t = \bar{y}$, i.e.,

$$\lim_{t\to\infty} \sup\{u_t, u_{t+1}, \ldots\} = \bar{y}$$

Clearly, lim inf and lim sup are both well-defined social welfare functions. These functions are entirely insensitive to the utility levels of the generations that are living at the present or in the "near future." Welfare comparisons using either of these criteria depend only on the utility levels of generations born in the distant future. Chichilnisky [9] pointed out that such criteria give a "dictatorial role" to the future. Formally, a social welfare function $W(.)$ is said to display "dictatorship of the future" if it has the following property:

Definition 2 (*Dictatorship of the future*). A social welfare function $W(.)$ is said to display "dictatorship of the future" if the following condition holds: for every pair $(\mathbf{u}^1, \mathbf{u}^2)$, $W(\mathbf{u}^1)$ is greater than $W(\mathbf{u}^2)$ if and only if for all T sufficiently large, $W({}_T\mathbf{a}^1, \mathbf{u}^1_T) > W({}_T\mathbf{a}^2, \mathbf{u}^2_T)$ for all pairs of sequences $(\mathbf{a}^1, \mathbf{a}^2)$, where $({}_T\mathbf{a}^i, \mathbf{u}^i_T)$ means that the first $T+1$ elements of $\mathbf{u}^i$ are replaced by the vector ${}_T\mathbf{a}^i \equiv (a^i_0, a^i_1, \ldots, a^i_T)$.

Both the lim inf and the lim sup social welfare functions display dictatorship of the future. A welfare function is said to display "non-dictatorship of the future" if there is a pair $(\mathbf{u}^1, \mathbf{u}^2)$ such that $W(\mathbf{u}^1) > W(\mathbf{u}^2)$ and one can find some modifications to the utilities of individuals in the early generations that would reverse the ranking, i.e., $W({}_T\mathbf{a}^1, \mathbf{u}^1_T) < W({}_T\mathbf{a}^2, \mathbf{u}^2_T)$ for some pair $(\mathbf{a}^1, \mathbf{a}^2)$ and for T sufficiently large.

Chichilnisky [9] argued that both dictatorship of the present and dictatorship of the future are undesirable. She proposed the following axioms:

Axiom 1. Non-dictatorship of the present.

Axiom 2. Non-dictatorship of the future.

Considering the space of all bounded utility sequences, Chichilnisky [9, Definition 6] defined a "*sustainable preference*" as a welfare criterion that satisfies Property 1 (completeness), Property 2 (strong Pareto), and Axioms 1 and 2. The Chichilnisky social welfare function, defined below, is an example of sustainable preference.

The welfare function she proposed is a weighted sum of two terms, the first term being the usual discounted stream of utilities, while the second term is defined in a way that its value depends only on the limiting behavior of the utility sequence.[18]

[17] More precisely, for all $T > \widehat{T}$ for some $\widehat{T}$ that may depend on u^1 and u^2.

[18] This welfare function has the strong Pareto property, and satisfies the axioms of "non-dictatorship of the present" and "non-dictatorship of the future." If two more axioms are added, "continuity" and "independence" (in the sense of linearity in u_t), then this is the only form the welfare function can take.

Formally,

$$W^C(\mathbf{u}^i) = (1-\theta)\sum_{t=0}^{\infty}\lambda_t u_t^i + \theta\phi(\mathbf{u}^i)$$

where $0<\theta<1$, $0<\lambda_t\leqslant 1$, $\sum_{t=0}^{\infty}\lambda_t<\infty$ and, by definition, $\phi(\mathbf{u}^i)\equiv \lim_{t\to\infty} u_t^i$. Here, the limit can be defined to be lim sup, or lim inf, or some weighted average of the two. The social welfare function $W^C(.)$ clearly has the properties of "non-dictatorship of the present" and "non-dictatorship of the future." The positive weight $(1-\theta)$ on discounted utilitarianism implies non-dictatorship of the future, and the positive weight θ on the limiting value of the utility sequence implies non-dictatorship of the present.

A major problem with the Chichilnsky welfare function $W^C(.)$ is that for many dynamic models, including the familiar one-sector growth model or the standard model of renewable resource extraction, there does not exist an optimal path under this objective function. The intuition behind this non-existence is as follows. The function $\phi(\mathbf{u}^i)=\lim_{t\to\infty}u_t^i$ would insist on reaching, in the long run, the golden rule stock, but does not care how soon or how early. The discounted utilitarianism part, $\sum_{t=1}^{\infty}\lambda_t u_t^i$, would insist on reaching, instead, the modified golden rule stock. Any path $\mathbf{u}^i$ that goes near the modified golden rule stock and eventually veers to the golden rule stock at some time T_i will be beaten by another path $\mathbf{u}^j$ that does a similar thing but at a later date $T_j>T_i$. The latter path $\mathbf{u}^j$ in turn will be beaten by another path $\mathbf{u}^h$ with $T_h>T_j$ and so on. So an optimal path does not exist.

We propose to modify the Chichilnsky criterion by discarding the second term, $\theta\phi(\mathbf{u}^i)$, and replacing it with the maximin utility.[19] The resulting social welfare function is denoted by W^{mbr}, where the superscript *mbr* stands for "mixed Bentham–Rawls:"

$$W^{mbr}(\mathbf{u}^i) = (1-\theta)\sum_{t=0}^{\infty}\beta^t u_t^i + \theta\inf\{u_0^i, u_1^i, \ldots, u_n^i, \ldots\} \tag{3}$$

where $0<\beta<1$. This social welfare function is a weighted average of two functions. The first function is the standard sum of discounted utilities. The second function is the Rawlsian part which places special emphasis on the utility of the least advantaged. The positive weight $(1-\theta)$ on the discounted utilitarianism part implies non-dictatorship of the future, just as it does for the Chichilnsky's welfare function.[20] We can also show that the positive weight θ on the Rawlsian part ensures non-dictatorship of the present.

Proposition 1. *The social welfare function W^{mbr} satisfies non-dictatorship of the present.*

Proof. For any T, no matter how large it is, let us consider the pair $(\mathbf{u}^1,\mathbf{u}^2)$, where $u_t^1=1$ and $u_t^2=1-\varepsilon$ for all $t\geqslant 0$, and where ε is positive and small enough to satisfy the following condition:

$$\varepsilon<\frac{\theta(1-\beta)}{2-\theta-\beta}$$

Then $W(\mathbf{u}^1)-W(\mathbf{u}^2)=(1-\theta)(\varepsilon/\beta)+\theta\varepsilon>0$. Now choose $\mathbf{a}^1=(0,0,0,\ldots)$ and $\mathbf{a}^2=(1,1,1,\ldots)$. Then

$$W({}_T\mathbf{u}^1,\mathbf{a}_T^1)-W({}_T\mathbf{u}^2,\mathbf{a}_T^2)=(1-\theta)\left[\sum_{t=0}^{T}\varepsilon\beta^t-\sum_{t=T+1}^{\infty}\beta^t\right]-\theta(1-\varepsilon)$$

$$=\frac{(1-\theta)(1-\beta^{T+1})\varepsilon}{1-\beta}-(1-\theta)\beta^{T+1}(1+\beta+\beta^2+\cdots)-\theta(1-\varepsilon)<0$$

Thus we have been able to reverse the ranking by modifying the tail ends of the two utility sequences. □

Proposition 2. *The mixed Bentham–Rawls criterion is a sustainable preference in the sense of Definition* 6 *of Chichilnisky.*

Proof. It is straightforward to verify that W^{mbr} displays completeness and strong Pareto. Non-dictatorship of the future follows from the positive weight $(1-\theta)$. Non-dictatorship of the present follows from Proposition 1. □

Let us offer a justification for our proposed social welfare function. As it is well known, the maximin principle leads to zero saving, no matter how small is the initial capital stock. Without capital accumulation, there is little chance that just institutions can be developed and sustained. To obtain reasonable results, the maximum principle must be modified: it must be supplemented by a just savings principle. Rawls [26,27] acknowledged that it is difficult to formulate a just savings principle. At the same time, certain reasonable assumptions would set limits on the savings rate. Thus, in dealing with intergenerational equity, Rawls proposed that the parties in the original situation are heads of family, and that the principle adopted must be such that the parties wish all earlier generations to have followed it. "Thus imagining themselves to be fathers, say, they are to ascertain how much they should set aside for their sons and grandsons by noting that they would believe themselves entitled to claim of their fathers and grandfathers" [27, p. 256]. The heads of family are "regarded as

[19] The maximin utility is inf, not lim inf.

[20] It also implies that social welfare is increasing in u_t, thus ensuring that Property 1 (strong Pareto) is satisfied. Thus the utility of the least advantaged is not the only thing that counts. One may say that this rules out "dictatorship of the least advantaged."

family lines, with ties of sentiment between successive generations" [26, p. 292]. A family line is at the same time "one" and "many." Being "one," it is like a single individual. There are no valid reasons to object to an individual's discounting of his future consumption. But a family line is also "many." The least advantaged individuals have special claims, not unlike those accorded to the "contemporaneous individuals" of the simpler atemporal Rawlsian model. It is therefore arguable that each contracting party would (i) place a special, non-discounted, weight on the utility level of the least advantaged generation of the family line, and at the same time (ii) care about the sum of weighted utilities of all generations. It seems also sensible to allow a trade-off between (i) and (ii) above, because each party represents a family line. Our proposed mixed Bentham–Rawls criterion is in sharp contrast to the standard utilitarian tradition (e.g. see any graduate macroeconomic textbook) which would treat a family line as an infinitely lived individual. Such a textbook position could result in requiring great sacrifices of early generations who are typically poor. In contrast, our proposed approach avoids imposing very high rates of savings at the earlier stages of accumulation. Our approach would seem to have the support of Rawls, who also complained about the standard utilitarian approach, as it "may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for the later ones that are far better off." He instead advocated that "when people are poor and savings are difficult, a lower rate of savings should be required; whereas in a wealthier society greater savings may reasonably be expected since the real burden is less" [26, p. 287].

Another justification for the MBR criterion is that it identifies an optimal path (unlike the Chichilnisky criterion, which does not), while at the same time satisfies three desirable "non-dictatorship" properties: non-dictatorship of the present, of the future, and of the least advantaged.

We now briefly mention some other properties of the MBR criterion. First, it is continuous in the supnorm topology.[21]As has been showed by Chichilnisky, the discounted utilitarian part is continuous in the supnorm topology. So is the function $\inf\{u_0^i, u_1^i, \ldots, u_n^i, \ldots\}$. The MBR criterion, being the sum of two continuous functions, is itself continuous in the supnorm topology.[22] Another property is that it does not satisfy Koopmans' stationarity postulate. This means that if the planner were allowed to re-plan at some future time $\tau \geqslant 1$, then he would use $\inf\{u_\tau^i, u_{\tau+1}^i, \ldots, u_n^i, \ldots\}$ instead of $\inf\{u_0^i, u_1^i, \ldots, u_n^i, \ldots\}$, and this would in general lead to time inconsistency. For example, if generation 0 is the least advantaged generation in the original plan, then the weights given, in the original plan, to generations g and h (where $h > g > 0$) are, respectively, $(1-\theta)\beta^g$ and $(1-\theta)\beta^h$. However, if at time $\tau = g$ the planner replans, and generation g is the least advantaged (compared with all other generations to follow), then the weights are $[(1-\theta)+\theta]$ for generation g and $(1-\theta)\beta^{h-g}$ for generation h. The relative weight of these two generations therefore changes. To deal with this problem, one would have to formulate a game-theoretic problem with a sequence of planners, each knowing that the subsequent one would use a different set of relative weights. One would then look for a Nash equilibrium of this game. This is a topic for future research.

We conjecture that in the familiar model of Dasgupta–Heal–Solow, sophisticated planning under the MBR criterion would lead to the eventual vanishing of consumption. Intuitively, maximizing the weighted sum of discounted utilities and a lower bound on consumption in the D–H–S model would lead to a path that reaches this level in finite time and stays there. Along this eventual constant consumption phase, the marginal product of capital goes to zero. Reapplication of the MBR criterion would lead to an initial consumption splurge and an eventual phase with a lower constant consumption. Thus, sophisticated planning implies that consumption goes to zero in the D–H–S model. The sophisticated path is identical to the discounted utilitarian path.

3. Finding the social optimum under the mixed Bentham–Rawls criterion

In this section we derive necessary conditions for an optimal path under the mixed Bentham–Rawls criterion. It is more convenient to work with a continuous time model. Let x be a vector of n state variables, and c a vector of m control variables. Denote the instantaneous utility function by $u(x(t), c(t), t)$.

The transition equations are $\dot{x}_i(t) = g_i(x(t), c(t), t)$, for $i = 1, 2, \ldots, n$. Given the values of the state variables, the control variables at time t must belong to a feasibility set $A(x(t), t)$ which is characterized by a set of s inequality constraints:

$$h_i(x(t), c(t), t) \geqslant 0, \quad i = 1, 2, \ldots, s \tag{4}$$

Consider first the case of a finite horizon T. The initial stocks $x_i(0)$, $i = 1, 2, \ldots, n$, are given. For a given time path $\widehat{c}(.)$ and the associated time path $\widehat{x}(.)$, let $\underline{u}$ be the greatest lower bound of the resulting time path of utility:

$$\underline{u} = \inf_t \{u(\widehat{x}(t), \widehat{c}(t), t)\}$$

[21] Formally, the space S of all utility sequences is a subset of the space of all infinite bounded sequences of real numbers, denoted by ℓ_∞. The supnorm is defined by

$$\|\mathbf{u}\| = \sup_{t=0,1,2,\ldots} |u_t|$$

[22] It is, however, not continuous in the topology of pointwise convergence.

This implies that

$$u(\widehat{x}(t), \widehat{c}(t), t) \geqslant \underline{u} \tag{5}$$

Assume a constant rate of discount $\rho \geqslant 0$. The social welfare generated by the time path $\widehat{c}(.)$ under the mixed Bentham–Rawls criterion is then the continuous time counterpart of (3):

$$\int_0^T (1-\theta)\, e^{-\rho t} u(\widehat{x}(t), \widehat{c}(t), t)\, dt + \theta\, \underline{u} \tag{6}$$

To maximize social welfare given the vector of initial stocks $x_0 \equiv (x_{10}, x_{20}, \ldots, x_{n0})$, the planner chooses the number $\underline{u}$ and the time path $c(t)$ to maximize the above welfare function, subject to (4) and (5) and

$$\dot{x}_i(t) = g_i(x(t), c(t), t) \tag{7}$$

and $x_i(T) \geqslant 0$.

Note that $\underline{u}$ must belong to a feasible set $Z(x_0)$, which is defined as follows:

$$Z(x_0) \equiv \left\{ \begin{array}{c} \widetilde{u} : \exists c(t) \in A(x(t), t) \quad \text{and} \quad u(x(t), c(t), t) \geqslant \widetilde{u}, \\ \text{subject to} \;\; \dot{x}(t) = g(x(t), c(t), t) \quad \text{and} \quad x(T) \geqslant 0, x(0) = x_0 \end{array} \right\} \tag{8}$$

We define

$$b \equiv \sup\{\widetilde{u} : \widetilde{u} \in Z(x_0)\} \tag{9}$$

Here b is the highest feasible minimum living standard that can be imposed as a constraint, given the initial stock x_0.

3.1. The necessary conditions

Since $\underline{u}$ is a constant to be chosen optimally, the optimization problem (6) is an optimal control problem with $\underline{u}$ treated as a control parameter. The necessary conditions for such problems can be derived from Hestenes's Theorem.[23] They are as follows.

Let $\pi(t)$ be the vector of costate variables, $\lambda(t)$ be the vector of multipliers associated with the inequality constraints (4) and $\omega(t)$ the multiplier associated with constraint (5). The Hamiltonian for this problem is

$$H(t, x(t), c(t), \pi(t)) \equiv (1-\theta)\, e^{-\rho t} u(x(t), c(t), t) + \pi(t) g(x(t), c(t), t)$$

and the Lagrangian is

$$L(t, x(t), c(t), \pi(t), \lambda(t), \omega(t), \underline{u}) = H + \lambda(t) h(x(t), c(t), t) + \omega(t)[u(x(t), c(t), t) - \underline{u}]$$

An optimal path must satisfy the following conditions:

(i) The maximum condition: the control variables maximize the Hamiltonian subject to the inequality constraints (4) and (5).
(ii) The adjoint equations: $\dot{\pi} = -\partial L / \partial x$.
(iii) The transition equations: $\dot{x} = \partial L / \partial \pi$.
(iv) The transversality condition for the optimal choice of the control parameter $\underline{u}$ is

$$\theta + \int_0^T \frac{\partial L}{\partial \underline{u}}\, dt \geqslant 0 \quad (= 0 \text{ if } \; \underline{u} < b)$$

(where b is defined by (9)), and the transversality condition for the optimal choice of the final stocks is $x(T) \geqslant 0$, $\pi(T) \geqslant 0$, $\pi(T) x(T) = 0$.
(v) The Hamiltonian and the Lagrangian are continuous functions of time, and, along the optimal path,[24]

$$\frac{d}{dt} H(t, x(t), c(t), \pi(t)) = \frac{d}{dt} L(t, x(t), c(t), \pi(t), \lambda(t), \omega(t), \underline{u}) = \frac{\partial L}{\partial t}$$

[23] See [21, Theorem 7.11.1] for an exposition of Hestenes' Theorem which deals with optimal control problems involving control parameters and various constraints.

[24] That is, the value of the total derivative of L (along the optimal path) equals the value of the partial derivative $\partial L / \partial t$, evaluated at the optimal vectors of controls, states, and multipliers.

3.2. Implications for genuine savings

Since one of the fundamental questions concerning intergenerational equity is "how much a generation ought to save" given its current circumstances, let us derive the implications of our mixed Bentham–Rawls criterion for saving rates. Following [16,17] we define "present-value genuine savings" by $S(t) \equiv \pi(t)g(x(t), c(t), t)$ and "current-value genuine savings" by $S^c(t) \equiv e^{\rho t}\pi(t)g(x(t), c(t), t)$. Then, by definition of the Hamitonian H and of genuine savings S,

$$\frac{d}{dt}H = -\rho(1-\theta)\,e^{-\rho t}u(t) + (1-\theta)\,e^{-\rho t}\dot{u} + \dot{S} \tag{10}$$

On the other hand,

$$\frac{\partial L}{\partial t} = -\rho(1-\theta)\,e^{-\rho t}u(t) + \pi(t)g_t + \lambda(t)h_t + [(1-\theta)\,e^{-\rho t} + \omega]u_t \tag{11}$$

Using (v), it follows that along the optimal path, utility is rising at time t if and only if the *rate of change* in present-value genuine savings, adjusted for the impact of technological progress (the term inside the curly brackets in the equation below), is negative:

$$\dot{S} - \{\pi g_t + \lambda h_t + [(1-\theta)\,e^{-\rho t} + \omega]u_t\} = -(1-\theta)\,e^{-\rho t}\dot{u} \tag{12}$$

i.e.,

$$\dot{S} + (1-\theta)\,e^{-\rho t}\dot{u} = \{\pi g_t + \lambda h_t + [(1-\theta)\,e^{-\rho t} + \omega]u_t\} \tag{13}$$

Thus, the constancy of present-value genuine savings ($\dot{S} = 0$) is consistent with growing utility if the impact of technological progress is positive. In particular, suppose the impact of technological progress is zero. Then, as is clear from (12), if utility is constant over some time interval $[t_1, t_2]$, then the present-value genuine savings must be constant as well. This result, known as the converse of Hartwick rule, was first established in a formal manner by Dixit et al. [12].

Now, by definition, $S(t) = e^{-\rho t}S^c(t)$. Hence

$$\frac{\dot{S}}{S} = -\rho + \frac{\dot{S}^c}{S^c}$$

Thus, under our objective function, along any time interval $[t_1, t_2]$ *when utility is constant, the current-value genuine savings, if positive, must be rising*:

$$\frac{\dot{S}^c}{S^c} = \rho \tag{14}$$

We will see that this result is confirmed in our numerical simulations below.

Remark. Eq. (13) is a generalization of results in [16,17]. Those papers were concerned only with the standard utilitarian objective, and thus had no place for the multiplier ω.

3.3. Infinite horizon optimization under the mixed Bentham–Rawls criterion

Suppose the time horizon is infinite and the rate of discount ρ is a positive constant. Then the social planner chooses $\underline{u}$ and $c(.)$ to maximize the mixed Rawls–Bentham objective function:

$$\theta\,\underline{u} + \int_0^\infty (1-\theta)u(x,c,t)\,e^{-\rho t}\,dt$$

It will be convenient to re-write this objective function as an integral:

$$\int_0^\infty \{\theta\,\underline{u}\,\rho + (1-\theta)u(x,c,t)\}\,e^{-\rho t}\,dt \tag{15}$$

Let $\psi(t) = e^{\rho t}\pi(t)$, $\mu(t) = e^{\rho t}\lambda(t)$ and $w(t) = e^{\rho t}\omega(t)$. The current-value Hamiltonian of this infinite horizon problem is $H^c = \theta\,\underline{u}\,\rho + (1-\theta)u(x,c,t) + \psi g(x,c,t)$ and the current-value Lagrangian is $L^c = H^c + \mu h(x,c,t) + w[u(x,c,t) - \underline{u}]$. Then we obtain the following conditions:

$$\frac{\partial L^c}{\partial c} = (1-\theta)u_c + \psi g_c + \mu h_c + w u_c = 0$$

$$\mu \geqslant 0, \quad g(x,c,t) \geqslant 0, \quad \mu g(x,c,t) = 0$$

$$w \geqslant 0, \quad u(x,c,t) - \underline{u} \geqslant 0, \quad w[u(x,c,t) - \underline{u}] = 0$$

$$\dot{\psi} = \rho\psi - \frac{\partial L^c}{\partial x}$$

$$\dot{x} = \frac{\partial L^c}{\partial \psi}$$

The optimality condition with respect to the control parameter is

$$\int_0^\infty e^{-\rho t} \frac{\partial L^c}{\partial \underline{u}}\, dt \geqslant 0 \quad (= 0 \text{ if } \underline{u} < \underline{u}_H)$$

where $\underline{u}_H$ is the infinite-horizon counterpart of b in Eq. (9). Finally, the transversality conditions with respect to the stocks are

$$\lim_{t\to\infty} e^{-\rho t}\psi(t) \geqslant 0 \quad \text{and} \quad \lim_{t\to\infty} e^{-\rho t}\psi(t)x(t) = 0$$

4. An example: optimal renewable resource use under the MBR criterion

Let us illustrate the implications of the MBR criterion for a simple model of renewable resource exploitation.[25] The resource stock is a scalar $x(t)$. Its growth function is $\dot{x} = G(x) - c$, where $G(x)$ is a strictly concave function which reaches a maximum at some $x_M > 0$. We call x_M the "maximum sustainable yield" stock level. Assume $G(0) = 0$ and $G'(0) > 0$. The variable c denotes the harvest rate.

The utility function is assumed to depend only on consumption: $u = u(c)$. We assume u to be increasing, $u'(c) > 0$, and concave $u''(c) < 0$. Furthermore we assume the standard Inada conditions, $\lim_{t\to\infty} u'(0) = \infty$ and $\lim_{t\to\infty} u'(\infty) = 0$, are satisfied.

We define the "golden rule stock level," denoted by x_g, as the stock level that maximizes long-run sustainable utility:

$$\max_x u(G(x))$$

This level is uniquely determined by the equation $u'(G(x_g))G'(x_g) = 0$. Since preferences depend only on consumption and utility is increasing in c, the "golden rule stock of level" coincides with the "maximum sustainable yield," $x_g = x_M$.

By the "modified golden rule stock level," we mean the stock level $\overline{x}_u$ which is defined by the equation $G'(\overline{x}_u) = \rho$, where ρ is the rate of time preference. Notice that $\overline{x}_u$ coincides with the steady-state stock of resource under the discounted utilitarian criterion. Since $G(x)$ is strictly concave, $\overline{x}_u < x_g$.

Now consider the optimal growth program under the mixed Bentham–Rawls objective function

$$\max \theta\, \underline{u} + (1-\theta) \int_0^\infty e^{-\rho t} u(c)\, dt$$

subject to $\dot{x} = G(x) - c$, $u(c) \geqslant \underline{u}$, where $x(0) = x_0 > 0$.

Under the MBR criterion, does the optimal path approach a steady state that is somewhere between the modified golden rule stock level, $\overline{x}_u$, and the golden rule stock level, x_g? The following proposition, whose proof can be found at JEEM's online archive of supplementary material (at http://www.eare.org/journal/index.html), gives the answer.

Proposition~3. *Under the MBR criterion, the steady state depends on whether the initial stock, x_0, is smaller or greater than $\overline{x}_u$.*

(i) *If $x_0 > \overline{x}_u$, the optimal path consists of two phases. Phase I begins at $t = 0$ and ends at some finite $t_\theta > 0$. During Phase I, the utility level and the resource stock are both falling. Genuine saving is negative and rises toward zero. At time t_θ, the pair (x, c) reaches a mixed Bentham–Rawls steady-state pair $(\overline{x}_{mbr}, \overline{c}_{mbr})$ where $\overline{x}_u < \overline{x}_{mbr} < x_g$. During Phase II, the system stays at the mixed Bentham–Rawls steady state $(\overline{x}_{mbr}, \overline{c}_{mbr})$. Genuine saving is constant and equal to zero.*

(ii) *If $x_0 < \overline{x}_u$, the optimal path also consists of two phases. Phase I begins at $t = 0$ and ends at some finite $t_\theta > 0$. During Phase I, utility is constant, which implies a time path of constant harvest rate and rising stock. Genuine saving in this phase is positive and increasing. In Phase II, the economy follows the standard utilitarian path approaching asymptotically the utilitarian steady state given by the modified golden rule stock level, $\overline{x}_u$. In this phase genuine saving falls steadily toward zero.*

Remark. The above solution is based on the assumption that no replanning is allowed. Consider part (i). If at some time $\tau \in (0, t_\theta)$ replanning takes place, then Phase I will be lengthened beyond t_θ, and the mixed Bentham–Rawls steady state $(\overline{x}_{mbr}, \overline{c}_{mbr})$ will be moved down along the curve $G(x)$, but $\overline{x}_{mbr}$ still remains higher than $\overline{x}_u$. Alternatively, consider part (ii). Suppose at some time $\tau > t_\theta$, Phase I has passed, and re-planning is then permitted. Then a new Phase I will begin again. To deal with this time-inconsistency, one would have to consider a game played by a sequence of planners, each knowing that subsequent planners would have an incentive to replan. One would look for a Markov-perfect equilibrium along the lines used in [2].

[25] This model is isomorphic to the standard one-sector model of exogenous growth so our results are readily applicable to that model.

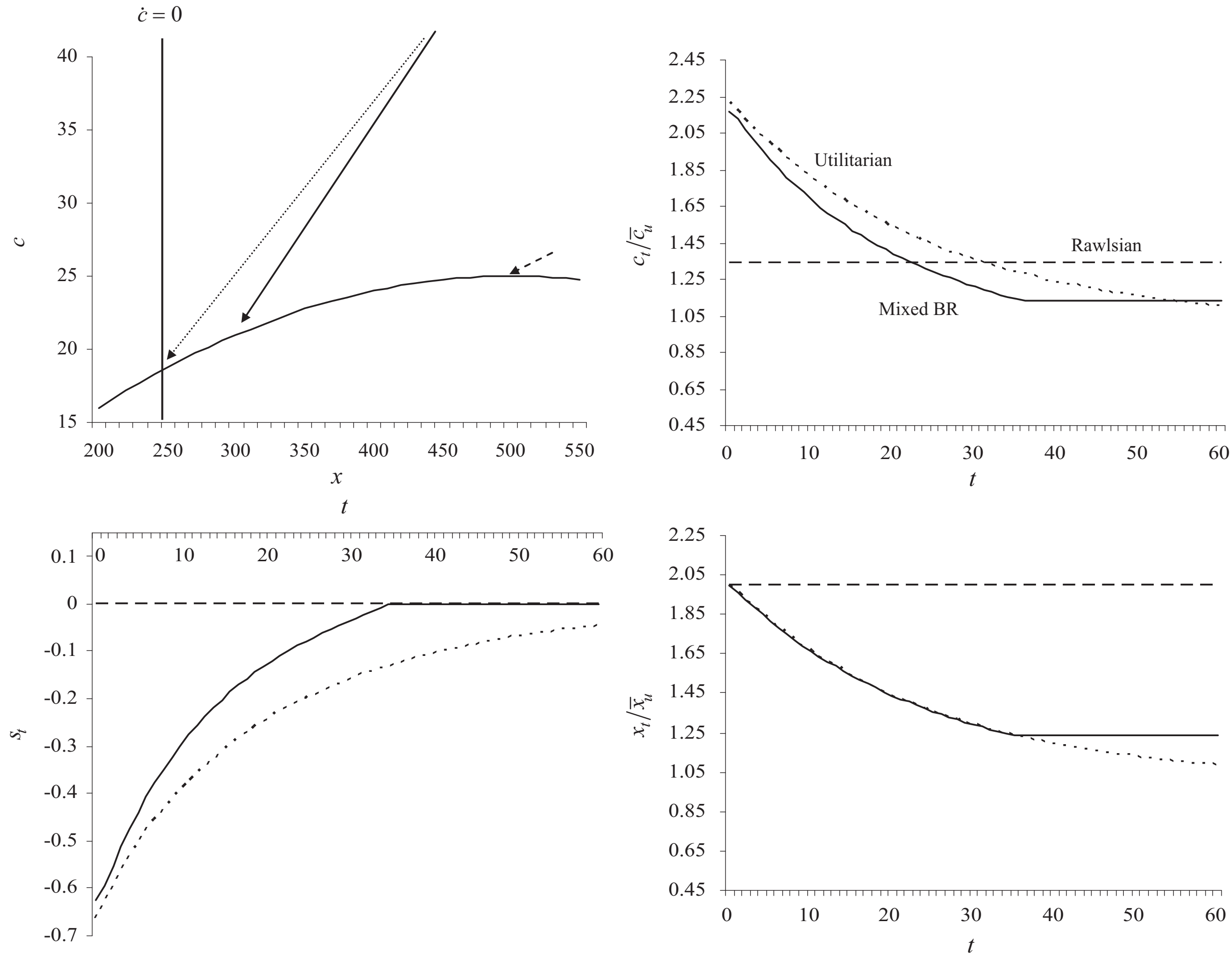

Fig. 1. Transitional dynamics: $x_0 = 3 * \bar{x}_u$.

5. Sensitivity analysis of optimal paths under the MBR criterion

We assume logarithmic preferences, $u(c) = \ln c$, and a logistic specification for the reproduction function of the resource:

$$\dot{x} = rx\left(1 - \frac{x}{K}\right) - c$$

We set the intrinsic regeneration rate, $r = 0.1$, and the carrying capacity, $K = 1000$, and a standard value for the rate of time preference, $\rho = 0.05$. Finally, we assign equal weights to the Rawlsian and utilitarian components in our objective function, so $\theta = 0.5$. Let us use subscripts u, r and mbr to denote the solutions under the discounted utilitarian, maximin and mixed Bentham–Rawls criteria, respectively, and denote steady states by upper-bar variables. Under our benchmark calibration the modified golden rule stock of resource, the utilitarian steady state, is $\bar{x}_u = 250$ and the golden rule stock of resource is $x_g = 500$.

As already pointed out, the dynamic adjustment and the steady state of our economy depend crucially on whether the initial resource stock, x_0, is below or above $\bar{x}_u$. We shall examine both cases in detail.

Fig. 1 illustrates the transitional dynamics of an economy that begins with a stock of resources above the golden rule stock, i.e., $x_0 = 3 * \bar{x}_u$. Panel (a) reproduces the phase diagram in the consumption-resource stock space under the three solution criteria. Since this initial stock is above the Rawlsian steady state we have imposed a constant path of consumption equal to golden rule level of consumption, $c_r(t) = \bar{c}_r$, for the Rawlsian solution.[26] This consumption path exceeds the

[26] Since the initial stock of resources is above the golden rule level, the Rawlsian criterion is consistent with any transition that guarantees at least the golden rule level of consumption. Given this we have assumed in our simulations that the stock of resources is slowly depleted by a constant path of consumption equal to the golden rule level.

Table 1
Sensitivity analysis optimal renewable resource extraction.

	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$x_0 = 2 * \bar{x}_u$	$x_0 = 4 * \bar{x}_u$
$\bar{x}_u$	–	250.00	–	–	–
$\bar{c}_u$	–	18.75	–	–	–
$c_u(0)/\bar{c}_u$	–	3.431		2.215	4.646
$\bar{x}_r/\bar{x}_u$	–	2.000	–	2.000	2.000
$\bar{c}_r/\bar{c}_u$	–	1.333	–	1.333	1.333
$\bar{x}_{mbr}/\bar{x}_u$	1.197	1.294	1.436	1.234	1.332
$\bar{c}_{mbr}/\bar{c}_u$	1.118	1.167	1.227	1.138	1.185
$\bar{c}_{mbr}(0)/\bar{c}_u$	3.517	3.360	3.257	2.106	4.382
t_θ	53.65	42.43	32.79	36.15	45.97

Benchmark: $\theta = 0.5$, $\rho = 0.05$, $r = 0.1$, $K = 1000$, $x_0 = 3 * \bar{x}_u$.

modified golden rule level by 33% guaranteeing that each future generation eventually inherits the golden rule stock of resources. On the other hand the utilitarian dynamics are driven by the trade-off between the marginal rate of growth of the resource and the rate of time preference. Since with the initial stock above the golden rule this trade-off is dominated by impatience, the initial generation enjoys a level of consumption that exceeds by almost 3.5 times the modified golden rule one. At this level, consumption is substantially larger than the capacity of regeneration of the resource and therefore the resource stock begins to fall. The rest of the transition is characterized by subsequent decreases in consumption and the stock of resource, with genuine savings increasing monotonically toward zero as the rate of regeneration of the resource increases toward ρ.

The mixed Bentham–Rawls criterion, which is a compromise between the previous two, leads to an optimal path with two clearly distinctive phases. During Phase I, which begins at $t = 0$ and ends at some finite time, $t_\theta > 0$, the solution follows the unstable dynamics of the utilitarian solution. The initial level of consumption lies between the Rawlsian choice and the utilitarian one, exceeding the modified golden rule level of consumption by a factor of 3.36. As in the utilitarian case, this high level of consumption leads to an initial phase characterized by decreases in the resource stock and consumption, with the saving rate monotonically increasing toward zero. This process of stock depletion continues for 42 generations,[27] $t_\theta = 42.43$. At this point in time the mixed Bentham–Rawls solution reaches its steady state characterized by a stock of resource that exceeds the modified golden rule stock by 29.4% representing around 65% of the Rawlsian steady-state stock of resource. The path of welfare mimics that of consumption. Since the stock of resource is initially high, the utilitarian solution yields an initial level of welfare that exceeds the modified golden rule level of welfare by 42%. As a result of this high rate of consumption the first 46 generations are better off under the utilitarian solution than under any of the other two. On the other hand the Rawlsian solution yields a constant path of welfare that exceeds by 10% that of the utilitarian steady state. The mixed Bentham–Rawls solution takes advantage of the initial abundance of resources to provide early generations with a level of welfare that exceeds the Rawlsian one by more than 25%, but at the same time uses this abundance to guarantee all future generations a level of welfare that exceeds the utilitarian steady state one by 5.2%.

Table 1 conducts some sensitivity analysis. As the weight placed in the Rawlsian component of our objective function, θ, increases the mixed Bentham–Rawls steady-state stock of resource increases toward the Rawlsian one. This reduces the level of consumption during the initial generations and shortens the length of the transition. As the initial stock of resources increases, the mixed Bentham–Rawls steady-state stock of resource increases relative to the utilitarian and Rawlsian ones, which still reach the modified golden rule and the golden rule stock of resource, respectively. The length of the initial phase of the transition also increases. In a sense the higher initial stock is fairly distributed across generations, on one hand increasing the number of generations in Phase I and on the other hand increasing the steady-state level of consumption to allow all future generations to enjoy higher levels of welfare.

Fig. 2 and Table 2 summarize our results when the initial stock of resources is below the modified golden rule stock, specifically in our benchmark calibration we consider the case when $x_0 = 0.5 * \bar{x}_u$. The maximin solution chooses a level of consumption below 60% of the modified golden rule level of consumption, consistent with a stock of resource that stays constant and equal to its initial level. The utilitarian solution with its emphasis on intertemporal trade-offs chooses a level of consumption for the initial generation that is only 39% of the modified golden rule level. This low level of initial consumption is associated with a high saving rate (34%) imposed on the poorest generation, and a substantial increases in the resource stock. The utilitarian transition is characterized by a monotonic increase in consumption and the stock of resource and a monotonic decrease in genuine saving. Our proposed criterion leads to an initial level of consumption that exceeds its utilitarian counterpart by 25%, still allowing for a rate of saving of 15%. During Phase I of the mixed Bentham–Rawls transition consumption remains constant as the stock of resource accumulates. Intuitively the Rawlsian component of our mixed Bentham–Rawls solution places a floor on the initial level of consumption that lies somewhere

[27] Notice that under our benchmark calibration the speed of convergence exhibited by our model is very low, with the half-life of a deviation close to 20 generations.

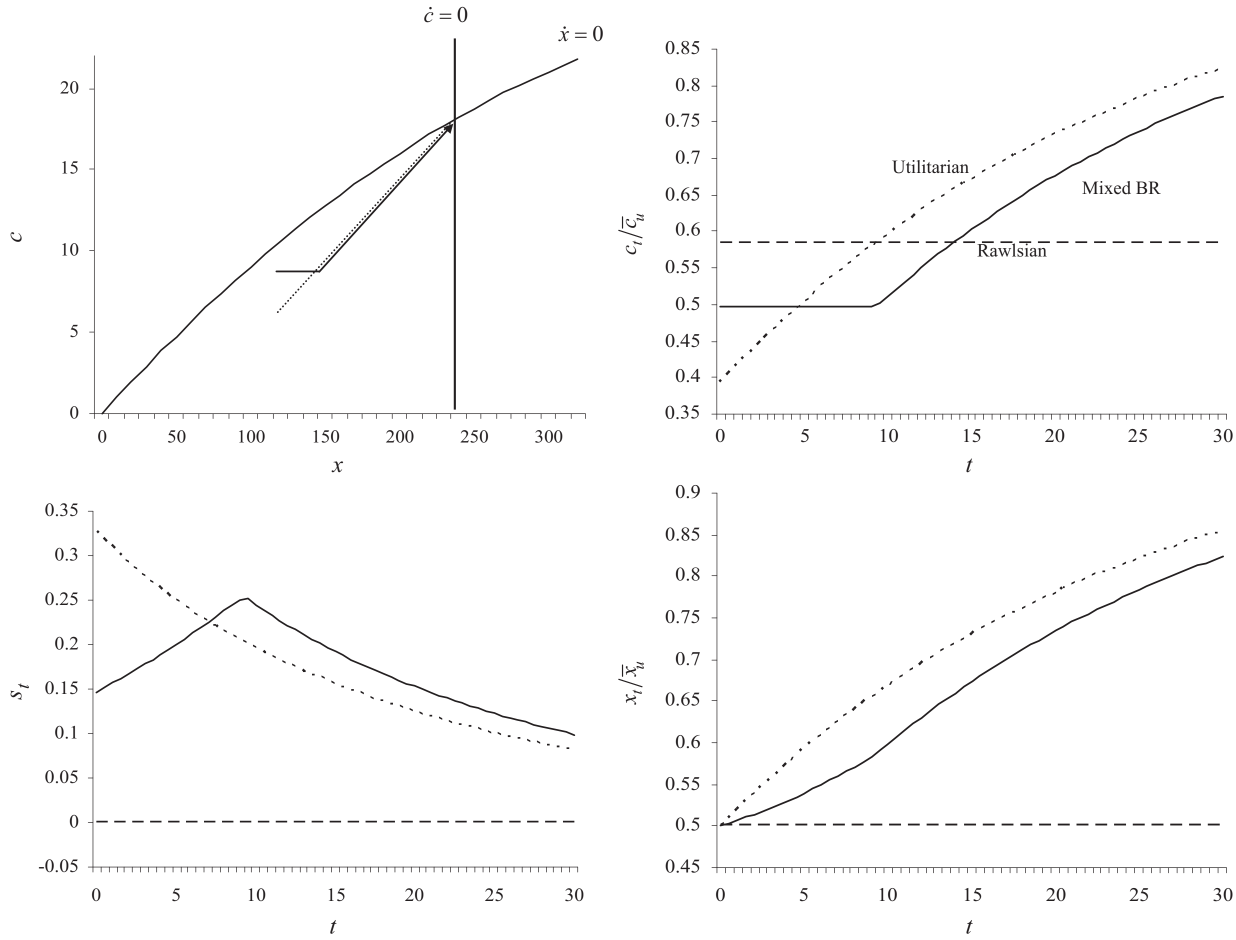

Fig. 2. Transitional dynamics: $x_0 = 0.5 * \bar{x}_u$.

Table 2
Sensitivity analysis optimal renewable resource extraction.

	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$x_0 = 0.25 * \bar{x}_u$	$x_0 = 0.75 * \bar{x}_u$
$\bar{x}_u$	–	250	–	–	–
$\bar{c}_u$	–	18.75	–	–	–
$c_u(0)/\bar{c}_u$	–	0.392		0.089	0.696
$\bar{x}_r/\bar{x}_u$	–	0.500	–	0.250	0.750
$\bar{c}_r/\bar{c}_u$	–	0.583	–	0.311	0.813
$c_\theta/\bar{c}_u$	0.459	0.498	0.531	0.197	0.773
t_θ	4.94	9.27	15.70	7.35	13.59

Benchmark: $\theta = 0.5$, $\rho = 0.05$, $r = 0.1$, $K = 1000$, $x_0 = 0.5 * \bar{x}_u$.

between the Rawlsian level and the utilitarian one. With consumption constant and the stock of resource increasing the saving rate increases during this phase. After nine generations, $t_\theta = 9.27$, the saving rate peaks and Phase II of the transition begins with consumption growing at a rate that still allows for positive saving and therefore increases in the stock of resource that monotonically converges to the modified golden rule stock. Thus if x_0 is below $\bar{x}_u$, the mixed Bentham–Rawls criteria might be explained intuitively as placing a floor under an initial phase of the consumption path.

Decreases in the initial stock of resources lead to simultaneous decreases in the initial level of consumption and in the length of Phase I of the transition under our proposed criterion. As we increase the weight placed on the Rawlsian component of our objective function, θ, the initial level of consumption, c_θ, and the length of Phase I of the transition, t_θ, increase. Fig. 3 reproduces the relation between the weight assigned by the mixed Bentham–Rawls criterion to the

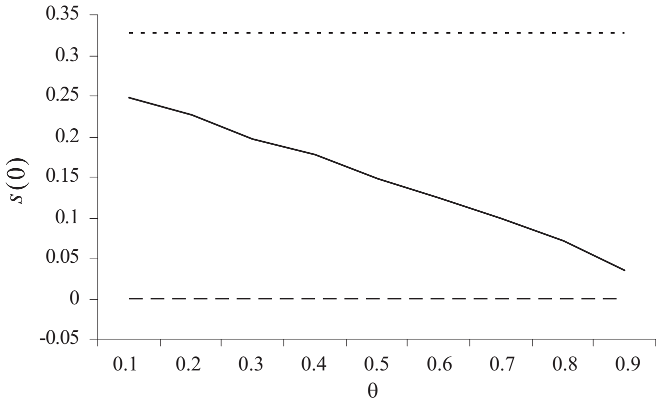

Fig. 3. Sensitivity of the initial saving rate to θ: $x_0 = 0.5 * \bar{x}_u$.

maximin component, θ, and the initial level of saving when $x_0 = 0.5 * \bar{x}_u$. As we increase the importance of the maximin component in our criterion the initial level of saving decreases converging to the Rawlsian choice in the limit when $\theta = 1$.

6. Concluding remarks

In this paper, we proposed a new welfare criterion, called the mixed Bentham–Rawls criterion, which we believe does justice to Rawls's nuanced notion of intergenerational equity. We have restricted attention to the problem of intergenerational equity, and to facilitate the analysis, we have abstracted from intra-generational equity.

We showed that optimal growth paths under the mixed Bentham–Rawls criterion can be characterized using standard techniques. These paths seem intuitively plausible, and reflect both the Rawlsian concerns for the least advantaged and the utilitarian principle. We also obtained a characterization of the relationship between the growth rate of genuine savings and the growth rate of utility.

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